Vietnam Team Selection Test 2010

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Day 1

1 Let n be a positive integer. Let T_n be a set of positive integers such that:

$$T_n = \{11(k+h) + 10(n^k + n^h) | (1 \le k, h \le 10)\}$$

Find all n for which there don't exist two distinct positive integers $a, b \in T_n$ such that $a \equiv b \pmod{110}$

2 Let ABC be a triangle with $\widehat{BAC} \neq 90^\circ$. Let M be the midpoint of BC. We choose a variable point D on AM. Let (O_1) and (O_2) be two circle pass through D and tangent to BC at B and C. The line BA and CA intersect $(O_1), (O_2)$ at P, Q respectively.

a) Prove that tangent line at P on (O_1) and Q on (O_2) must intersect at S.

b) Prove that S lies on a fix line.

3 We call a rectangle of the size 1×2 a domino. Rectangle of the 2×3 removing two opposite (under center of rectangle) corners we call tetramino. These figures can be rotated.

It requires to tile rectangle of size 2008×2010 by using dominoes and tetraminoes. What is the minimal number of dominoes should be used?

Day 2

1 Let a, b, c be positive integers which satisfy the condition: $16(a+b+c) \ge \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.
Prove that

$$\sum_{cyc} \left(\frac{1}{a+b+\sqrt{2a+2c}} \right)^3 \le \frac{8}{9}$$

2 We have n countries. Each country have m persons who live in that country ($n > m > 1$). We divide $m \cdot n$ persons into n groups each with m members such that there don't exist two persons in any groups who come from one country.
Prove that one can choose n people into one class such that they come from different groups and different countries.

3 Let S_n be sum of squares of the coefficient of the polynomial $(1+x)^n$. Prove that $S_{2n} + 1$ is not divisible by 3.