National Science Olympiad 2017

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–	Day 1
1	$ABCD$ is a parallelogram. g is a line passing A. Prove that the distance from C to g is either the sum or the difference of the distance from B to g, and the distance from D to g.
2	Five people are gathered in a meeting. Some pairs of people shakes hands. An ordered triple of people (A, B, C) is a *trio* if one of the following is true: -A shakes hands with B, and B shakes hands with C, or -A doesn't shake hands with B, and B doesn't shake hands with C. If we consider (A, B, C) and (C, B, A) as the same trio, find the minimum possible number of trios.
3	A positive integer d is *special* if every integer can be represented as $a^2 + b^2 - dc^2$ for some integers a, b, c. -Find the smallest positive integer that is not special. -Prove 2017 is special.
4	Determine all pairs of *distinct* real numbers (x, y) such that both of the following are true: -$x^{100} - y^{100} = 2^{99}(x - y)$ -$x^{200} - y^{200} = 2^{199}(x - y)$
–	Day 2
5	A polynomial P has integral coefficients, and it has at least 9 different integral roots. Let n be an integer such that $\|P(n)\| < 2017$. Prove that $P(n) = 0$.
6	Find the number of positive integers n not greater than 2017 such that n divides $20^n + 17k$ for some positive integer k.
7	Let $ABCD$ be a parallelogram. E and F are on BC, CD respectively such that the triangles ABE and BCF have the same area. Let BD intersect AE, AF at M, N respectively. Prove

there exists a triangle whose side lengths are BM, MN, ND.

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8 A field is made of 2017×2017 unit squares. Luffy has k gold detectors, which he places on some of the unit squares, then he leaves the area. Sanji then chooses a 1500×1500 area, then buries a gold coin on each unit square in this area and none other. When Luffy returns, a gold detector beeps if and only if there is a gold coin buried underneath the unit square it's on. It turns out that by an appropriate placement, Luffy will always be able to determine the 1500×1500 area containing the gold coins by observing the detectors, no matter how Sanji places the gold coins. Determine the minimum value of k in which this is possible.
