Tuymaada Olympiad 2015

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–	Senior League
–	**Day 1**
1	On the football training there was n footballers - forwards and goalkeepers. They made k goals. Prove that main trainer can give for every footballer squad number from 1 to n such, that for every goal the difference between squad number of forward and squad number of goalkeeper is more than $n-k$. *(S. Berlov)*
2	D is midpoint of AC for $\triangle ABC$. Bisectors of $\angle ACB, \angle ABD$ are perpendicular. Find max value for $\angle BAC$ *(S. Berlov)*
3	$P(x,y)$ is polynomial with real coefficients and $P(x+2y,x+y)=P(x,y)$. Prove that exists polynomial $Q(t)$ such that $P(x,y)=Q((x^2-2y^2)^2)$ *A. Golovanov*
4	Let $n!=ab^2$ where a is free from squares. Prove, that for every $\epsilon>0$ for every big enough n it is true, that $$2^{(1-\epsilon)n}<a<2^{(1+\epsilon)n}$$ *M. Ivanov*
–	**Day 2**
5	There is some natural number $n>1$ on the board. Operation is adding to number on the board it maximal non-trivial divisor. Prove, that after some some operations we get number, that is divisible by 3^{2000} *A. Golovanov*
6	Let $0 \leq b \leq c \leq d \leq a$ and $a>14$ are integers. Prove, that there is such natural n that can not be represented as $$n=x(ax+b)+y(ay+c)+z(az+d)$$ where x,y,z are some integers.

K. Kohas

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7 In $\triangle ABC$ points M, O are midpoint of AB and circumcenter. It is true, that $OM = R - r$. Bisector of external $\angle A$ intersect BC at D and bisector of external $\angle C$ intersect AB at E. Find possible values of $\angle CED$

D. Shiryaev

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8 There are $\frac{k(k+1)}{2} + 1$ points on the planes, some are connected by disjoint segments (also point can not lies on segment, that connects two other points). It is true, that plane is divided to some parallelograms and one infinite region. What maximum number of segments can be drawn ?

A.Kupavski, A. Polyanski

– Junior League

– **Day 1**

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1 There are 100 different real numbers. Prove, that we can put it in 10×10 table, such that difference between two numbers in cells with common side are not equals 1

A. Golovanov

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2 We call number as funny if it divisible by sum its digits $+1$.(for example $1 + 2 + 1|12$,so 12 is funny) What is maximum number of consecutive funny numbers ?

O. Podlipski

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3 Same as Senior Problem 2

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4 Prove that there exists a positive integer n such that in the decimal representation of each of the numbers $\sqrt{n}$, $\sqrt[3]{n}, \ldots,$ $\sqrt[10]{n}$ digits 2015 stand immediately after the decimal point.

A.Golovanov

– **Day 2**

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5 Same as Senior Problem 5

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6 Is there sequence (a_n) of natural numbers, such that differences $\{a_{n+1} - a_n\}$ take every natural value and only one time and differences $\{a_{n+2} - a_n\}$ take every natural value greater 2015 and only one time ?

A. Golovanov

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7 CL is bisector of $\angle C$ of ABC and intersect circumcircle at K. I - incenter of ABC. $IL = LK$. Prove, that $CI = IK$

D. Shiryaev

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8 Four sages stand around a non-transparent baobab. Each of the sages wears red, blue, or green hat. A sage sees only his two neighbors. Each of them at the same time must make a guess about the color of his hat. If at least one sage guesses correctly, the sages win. They could consult before the game started. How should they act to win?
