AMC 10 2006

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by joml88, chess64, Mrdavid445, solafidefarms, rrusczyk

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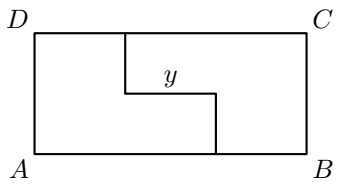
1	Sandwiches at Joe's Fast Food cost $\$3$ each and sodas cost $\$2$ each. How many dollars will it cost to purchase 5 sandwiches and 8 sodas? **(A)** 31 **(B)** 32 **(C)** 33 **(D)** 34 **(E)** 35
2	Define $x \otimes y = x^3 - y$. What is $h \otimes (h \otimes h)$? **(A)** $-h$ **(B)** 0 **(C)** h **(D)** $2h$ **(E)** h^3
3	The ratio of Mary's age to Alice's age is $3 : 5$. Alice is 30 years old. How old is Mary? **(A)** 15 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 50
4	A digital watch displays hours and minutes with cAM and cPM. What is the largest possible sum of the digits in the display? **(A)** 17 **(B)** 19 **(C)** 21 **(D)** 22 **(E)** 23
5	Doug and Dave shared a pizza with 8 equally-sized slices. Doug wanted a plain pizza, but Dave wanted anchovies on half the pizza. The cost of a plain pizza was $\$8$, and there was an additional cost of $\$2$ for putting anchovies on one half. Dave ate all the slices of anchovy pizza and one plain slice. Doug ate the remainder. Each paid for what he had eaten. How many more dollars did Dave pay than Doug? **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5
6	What non-zero real value for x satisfies $(7x)^{14} = (14x)^7$? **(A)** $\frac{1}{7}$ **(B)** $\frac{2}{7}$ **(C)** 1 **(D)** 7 **(E)** 14
7	The 8×18 rectangle $ABCD$ is cut into two congruent hexagons, as shown, in such a way that the two hexagons can be repositioned without overlap to form a square. What is y?  **(A)** 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

8 A parabola with equation $y = x^2 + bx + c$ passes through the points $(2, 3)$ and $(4, 3)$. What is c?
(A) 2 **(B)** 5 **(C)** 7 **(D)** 10 **(E)** 11

9 How many sets of two or more consecutive positive integers have a sum of 15?
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

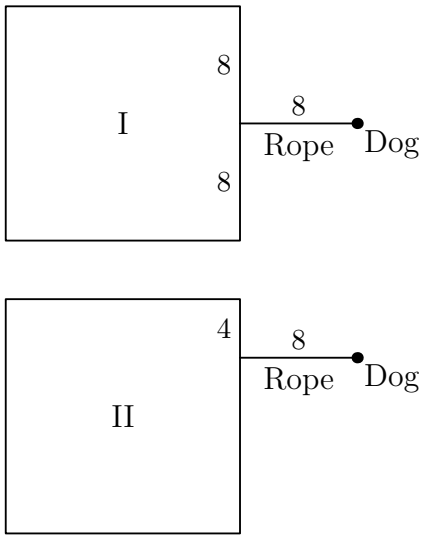
10 For how many real values of x is $\sqrt{120 - \sqrt{x}}$ an integer?
(A) 3 **(B)** 6 **(C)** 9 **(D)** 10 **(E)** 11

11 Which of the following describes the graph of the equation $(x + y)^2 = x^2 + y^2$?

(A) the empty set **(B)** one point **(C)** two lines

(D) a circle **(E)** the entire plane

12 Rolly wishes to secure his dog with an 8-foot rope to a square shed that is 16 feet on each side. His preliminary drawings are shown. Which of these arrangements gives the dog the greater area to roam, and by how many square feet?



(A) I, by 8π **(B)** I, by 6π **(C)** II, by 4π **(D)** II, by 8π **(E)** II, by 10π

13 A player pays $\$5$ to play a game. A die is rolled. If the number on the die is odd, the game is lost. If the number on the die is even, the die is rolled again. In this case the player wins if the second number matches the first and loses otherwise. How much should the player win if the game is fair? (In a fair game the probability of winning times the amount won is what the player should

pay.)
(A) $12 **(B)** $30 **(C)** $50 **(D)** $60 **(E)** $100

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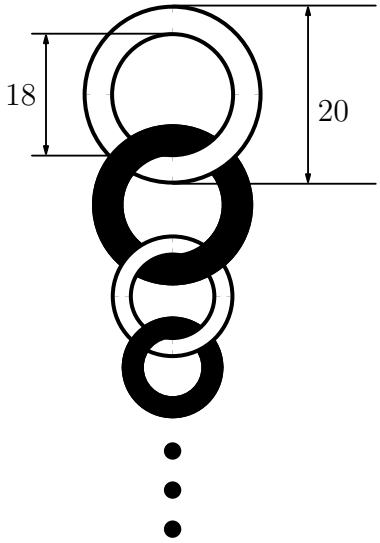
14 A number of linked rings, each 1 cm thick, are hanging on a peg. The top ring has an outside diameter of 20 cm. The outside diameter of each of the outer rings is 1 cm less than that of the ring above it. The bottom ring has an outside diameter of 3 cm. What is the distance, in cm, from the top of the top ring to the bottom of the bottom ring?



(A) 171 **(B)** 173 **(C)** 182 **(D)** 188 **(E)** 210

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15 Odell and Kershaw run for 30 minutes on a circular track. Odell runs clockwise at 250 m/min and uses the inner lane with a radius of 50 meters. Kershaw runs counterclockwise at 300 m/min and uses the outer lane with a radius of 60 meters, starting on the same radial line as Odell. How many times after the start do they pass each other?
(A) 29 **(B)** 42 **(C)** 45 **(D)** 47 **(E)** 50

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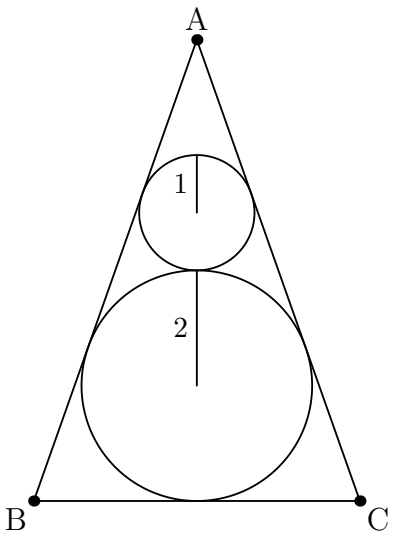
16 A circle of radius 1 is tangent to a circle of radius 2. The sides of $\triangle ABC$ are tangent to the circles as shown, and the sides $\overline{AB}$ and $\overline{AC}$ are congruent. What is the area of $\triangle ABC$?



(A) $\frac{35}{2}$ **(B)** $15\sqrt{2}$ **(C)** $\frac{64}{3}$ **(D)** $16\sqrt{2}$ **(E)** 24

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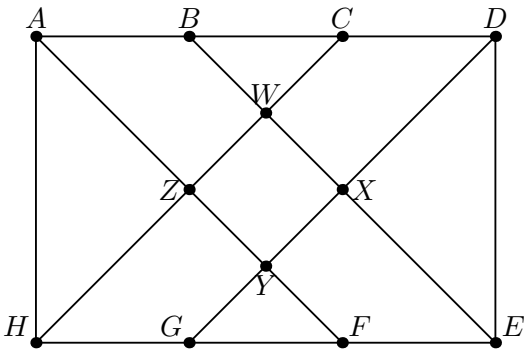
17 In rectangle $ADEH$, points B and C trisect $\overline{AD}$, and points G and F trisect $\overline{HE}$. In addition, $AH = AC = 2$. What is the area of quadrilateral $WXYZ$ shown in the figure?



(A) $\frac{1}{2}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** $\frac{2\sqrt{2}}{3}$ **(E)** $\frac{2\sqrt{3}}{3}$

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18 A license plate in a certain state consists of 4 digits, not necessarily distinct, and 2 letters, also not necessarily distinct. These six characters may appear in any order, except that the two letters must appear next to each other. How many distinct license plates are possible?
(A) $10^4 \cdot 26^2$ **(B)** $10^3 \cdot 26^3$ **(C)** $5 \cdot 10^4 \cdot 26^2$ **(D)** $10^2 \cdot 26^4$
(E) $5 \cdot 10^3 \cdot 26^3$

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19 How many non-similar triangle have angles whose degree measures are distinct positive inte-

gers in arithmetic progression?
(A) 0 **(B)** 1 **(C)** 59 **(D)** 89 **(E)** 178

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20 Six distinct positive integers are randomly chosen between 1 and 2006, inclusive. What is the probability that some pair of these integers has a difference that is a multiple of 5?
(A) $\frac{1}{2}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{4}{5}$ **(E)** 1

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21 How many four-digit positive integers have at least one digit that is a 2 or a 3?
(A) 2439 **(B)** 4096 **(C)** 4903 **(D)** 4904 **(E)** 5416

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22 Two farmers agree that pigs are worth $\$300$ and that goats are worth $\$210$. When one farmer owes the other money, he pays the debt in pigs or goats, with "change" received in the form of goats or pigs as necessary. (For example, a $\$390$ debt could be paid with two pigs, with one goat received in change.) What is the amount of the smallest positive debt that can be resolved in this way?
(A) $\$5$ **(B)** $\$10$ **(C)** $\$30$ **(D)** $\$90$ **(E)** $\$210$

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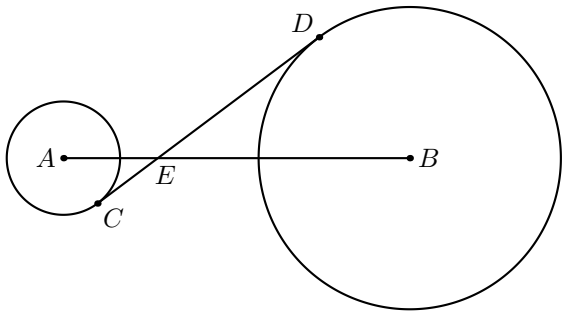
23 Circles with centers A and B have radii 3 and 8, respectively. A common internal tangent intersects the circles at C and D, respectively. Lines AB and CD intersect at E, and $AE = 5$. What is CD?



(A) 13 **(B)** $\frac{44}{3}$ **(C)** $\sqrt{221}$ **(D)** $\sqrt{255}$ **(E)** $\frac{55}{3}$

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24 Centers of adjacent faces of a unit cube are joined to form a regular octahedron. What is the volume of this octahedron?
(A) $\frac{1}{8}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

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25 A bug starts at one vertex of a cube and moves along the edges of the cube according to the following rule. At each vertex the bug will choose to travel along one of the three edges emanating from that vertex. Each edge has equal probability of being chosen, and all choices are independent. What is the probability that after seven moves the bug will have visited every vertex exactly once?

(A) $\frac{1}{2187}$ **(B)** $\frac{1}{729}$ **(C)** $\frac{2}{243}$ **(D)** $\frac{1}{81}$ **(E)** $\frac{5}{243}$

– B

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1 What is $(-1)^1+(-1)^2+\cdots+(-1)^{2006}$?

(A) -2006 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2006

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2 For real numbers x and y, define $x\spadesuit y=(x+y)(x-y)$. What is $3\spadesuit(4\spadesuit5)$?

(A) -72 **(B)** -27 **(C)** -24 **(D)** 24 **(E)** 72

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3 A football game was played between two teams, the Cougars and the Panthers. The two teams scored a total of 34 points, and the Cougars won by a margin of 14 points. How many points did the Panthers score?

(A) 10 **(B)** 14 **(C)** 17 **(D)** 20 **(E)** 24

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4 Circles of diameter 1 inch and 3 inches have the same center. The smaller circle is painted red, and the portion outside the smaller circle and inside the larger circle is painted blue. What is the ratio of the blue-painted area to the red-painted area?

(A) 2 **(B)** 3 **(C)** 6 **(D)** 8 **(E)** 9

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5 A 2 x 3 rectangle and a 3 x 4 rectangle are contained within a square without overlapping at any interior point, and the sides of the square are parallel to the sides of the two given rectangles. What is the smallest possible area of the square?

(A) 16 **(B)** 25 **(C)** 36 **(D)** 49 **(E)** 64

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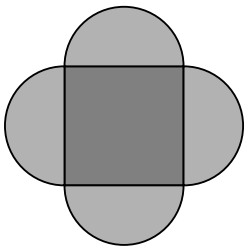
6 A region is bounded by semicircular arcs constructed on the side of a square whose sides measure $2/\pi$, as shown. What is the perimeter of this region?



(A) $\frac{4}{\pi}$ **(B)** 2 **(C)** $\frac{8}{\pi}$ **(D)** 4 **(E)** $\frac{16}{\pi}$

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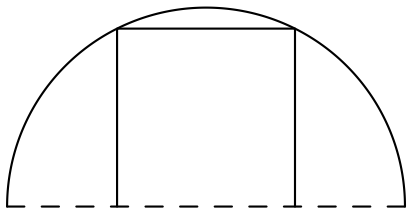
7 Which of the following is equivalent to $\sqrt{\frac{x}{1-\frac{x-1}{x}}}$ when $x<0$?

(A) $-x$ **(B)** x **(C)** 1 **(D)** $\sqrt{\frac{x}{2}}$ **(E)** $x\sqrt{-1}$

8 A square of area 40 is inscribed in a semicircle as shown. What is the area of the semicircle?



(A) 20π **(B)** 25π **(C)** 30π **(D)** 40π **(E)** 50π

9 Francesca uses 100 grams of lemon juice, 100 grams of sugar, and 400 grams of water to make lemonade. There are 25 calories in 100 grams of lemon juice and 386 calories in 100 grams of sugar. Water contains no calories. How many calories are in 200 grams of her lemonade?

(A) 129 **(B)** 137 **(C)** 174 **(D)** 223 **(E)** 411

10 In a triangle with integer side lengths, one side is three times as long as a second side, and the length of the third side is 15. What is the greatest possible perimeter of the triangle?

(A) 43 **(B)** 44 **(C)** 45 **(D)** 46 **(E)** 47

11 What is the tens digit in the sum $7! + 8! + 9! + \cdots + 2006!$?

(A) 1 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 9

12 The lines $x = \frac{1}{4}y + a$ and $y = \frac{1}{4}x + b$ intersect at the point $(1, 2)$. What is $a + b$?

(A) 0 **(B)** $\frac{3}{4}$ **(C)** 1 **(D)** 2 **(E)** $\frac{9}{4}$

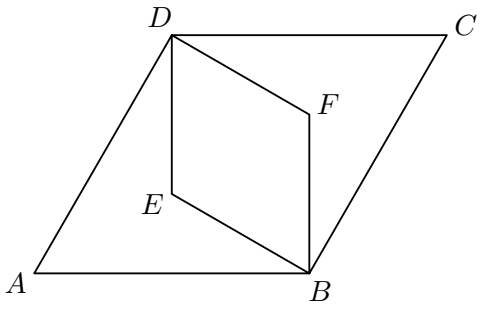
13 Joe and JoAnn each bought 12 ounces of coffee in a 16-ounce cup. Joe drank 2 ounces of his coffee and then added 2 ounces of cream. JoAnn added 2 ounces of cream, stirred the coffee well, and then drank 2 ounces. What is the resulting ratio of the amount of cream in Joe's coffee to that in JoAnn's coffee?

(A) $\frac{6}{7}$ **(B)** $\frac{13}{14}$ **(C)** 1 **(D)** $\frac{14}{13}$ **(E)** $\frac{7}{6}$

14 Let a and b be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that $a + (1/b)$ and $b + (1/a)$ are the roots of the equation $x^2 - px + q = 0$. What is q?

(A) $\frac{5}{2}$ **(B)** $\frac{7}{2}$ **(C)** 4 **(D)** $\frac{9}{2}$ **(E)** 8

15 Rhombus $ABCD$ is similar to rhombus $BFDE$. The area of rhombus $ABCD$ is 24, and $\angle BAD = 60^\circ$. What is the area of rhombus $BFDE$?



(A) 6 **(B)** $4\sqrt{3}$ **(C)** 8 **(D)** 9 **(E)** $6\sqrt{3}$

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16 Leap Day, February 29, 2004, occurred on a Sunday. On what day of the week will Leap Day, February 29, 2020, occur?

(A) Tuesday **(B)** Wednesday **(C)** Thursday **(D)** Friday **(E)** Saturday

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17 Bob and Alice each have a bag that contains one ball of each of the colors blue, green, orange, red, and violet. Alice randomly selects one ball from her bag and puts it into Bob's bag. Bob then randomly selects one ball from his bag and puts it into Alice's bag. What is the probability that after this process, the contents of the two bags are the same?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

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18 Let $a_1, a_2, \ldots$ be a sequence for which

$$a_1 = 2 \quad a_2 = 3 \quad \text{and} \quad a_n = \frac{a_{n-1}}{a_{n-2}} \text{ for each positive integer } n \geq 3.$$

What is a_{2006}?

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** 3

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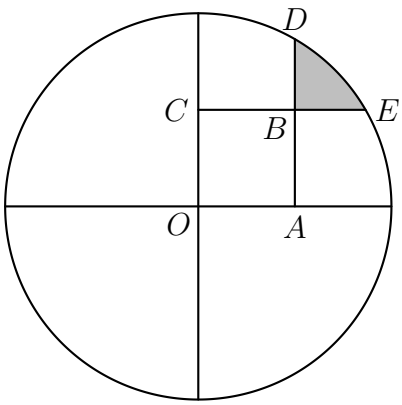
19 A circle of radius 2 is centered at O. Square $OABC$ has side length 1. Sides $\overline{AB}$ and $\overline{CB}$ are extended past b to meet the circle at D and E, respectively. What is the area of the shaded region in the figure, which is bounded by $\overline{BD}$, $\overline{BE}$, and the minor arc connecting D and E?



(A) $\frac{\pi}{3}+1-\sqrt{3}$ **(B)** $\frac{\pi}{2}\left(2-\sqrt{3}\right)$ **(C)** $\pi\left(2-\sqrt{3}\right)$ **(D)** $\frac{\pi}{6}+\frac{\sqrt{3}-1}{2}$
(E) $\frac{\pi}{3}-1+\sqrt{3}$

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20 In rectangle $ABCD$, we have $A=(6,-22)$, $B=(2006,178)$, and $D=(8,y)$, for some integer y. What is the area of rectangle $ABCD$?
(A) 4000 **(B)** 4040 **(C)** 4400 **(D)** $40,000$ **(E)** $40,400$

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21 For a particular peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5 and 6 on each die are in the ratio $1:2:3:4:5:6$. What is the probability of rolling a total of 7 on the two dice?
(A) $\frac{4}{63}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{8}{63}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{2}{7}$

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22 Elmo makes N sandwiches for a fundraiser. For each sandwich he uses B globs of peanut butter at 4 cents per glob and J blobs of jam at 5 cents per glob. The cost of the peanut butter and jam to make all the sandwiches is $\$2.53$. Assume that B, J, and N are all positive integers with $N>1$. What is the cost of the jam Elmo uses to make the sandwiches?
(A) $\$1.05$ **(B)** $\$1.25$ **(C)** $\$1.45$ **(D)** $\$1.65$ **(E)** $\$1.85$

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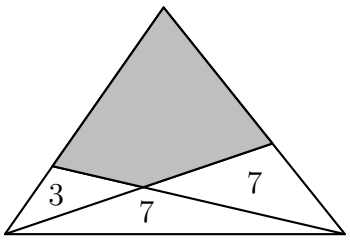
23 A triangle is partitioned into three triangles and a quadrilateral by drawing two lines from vertices to their opposite sides. The areas of the three triangles are 3, 7, and 7, as shown. What is the area of the shaded quadrilateral?



(A) 15 **(B)** 17 **(C)** $\frac{35}{2}$ **(D)** 18 **(E)** $\frac{55}{3}$

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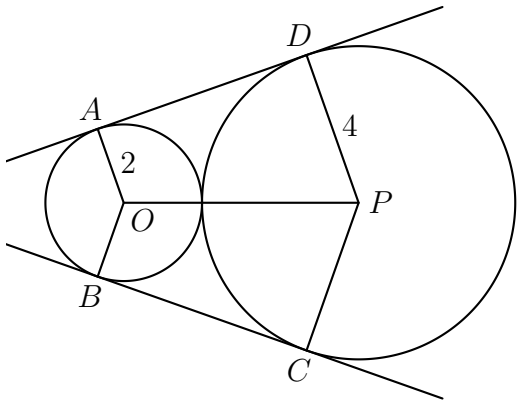
24 Circles with centers O and P have radii 2 and 4, respectively, and are externally tangent. Points A and B are on the circle centered at O, and points C and D are on the circle centered at P, such that $\overline{AD}$ and $\overline{BC}$ are common external tangents to the circles. What is the area of hexagon $AOBCPD$?



(A) $18\sqrt{3}$ **(B)** $24\sqrt{2}$ **(C)** 36 **(D)** $24\sqrt{3}$ **(E)** $32\sqrt{2}$

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25 Mr. Jones has eight children of different ages. On a family trip his oldest child, who is 9, spots a license plate with a 4-digit number in which each of two digits appears two times. "Look, daddy!" she exclaims. "That number is evenly divisible by the age of each of us kids!" "That's right," replies Mr. Jones, "and the last two digits just happen to be my age." Which of the following is not the age of one of Mr. Jones's children?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

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