

AMC 10 2008

www.artofproblemsolving.com/community/c4806

by worthawholebean, mathgeniuse^{ln(x)}, 123456789, Altheman, buzzer11, redcomet46, E^(pi*i)=-1, infinity4ever, funmath, azjps, resurrection, archimedes1, andersonw, nickster08, TZF, CatalystOfNostalgia, cognos599, undefined117, tenniskidperson3, n0vad3m0n, krsattack, rrusczyk

– A

– February 10th

1 A bakery owner turns on his doughnut machine at 8:30 AM. At 11:10 AM the machine has completed one third of the day's job. At what time will the doughnut machine complete the job?

(A) 1:50 PM (B) 3:00 PM (C) 3:30 PM (D) 4:30 PM (E) 5:50 PM

2 A square is drawn inside a rectangle. The ratio of the width of the rectangle to a side of the square is 2 : 1. The ratio of the rectangle's length to its width is 2 : 1. What percent of the rectangle's area is inside the square?

(A) 12.5 (B) 25 (C) 50 (D) 75 (E) 87.5

3 For the positive integer n , let $\langle n \rangle$ denote the sum of all the positive divisors of n with the exception of n itself. For example, $\langle 4 \rangle = 1 + 2 = 3$ and $\langle 12 \rangle = 1 + 2 + 3 + 4 + 6 = 16$ What is $\langle \langle \langle 6 \rangle \rangle \rangle$?

(A) 6 (B) 12 (C) 24 (D) 32 (E) 36

4 Suppose that $\frac{2}{3}$ of 10 bananas are worth as much as 8 oranges. How many oranges are worth as much as $\frac{1}{2}$ of 5 bananas?

(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) 4

5 Which of the following is equal to the product

$$\frac{8}{4} \cdot \frac{12}{8} \cdot \frac{16}{12} \cdots \frac{4n+4}{4n} \cdots \frac{2008}{2004}?$$

(A) 251 (B) 502 (C) 1004 (D) 2008 (E) 4016

6 A triathlete competes in a triathlon in which the swimming, biking, and running segments are all of the same length. The triathlete swims at a rate of 3 kilometers per hour, bikes at a rate of 20 kilometers per hour, and runs at a rate of 10 kilometers per hour. Which of the following is closest to the triathlete's average speed, in kilometers per hour, for the entire race?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

- 7 The fraction

$$\frac{(3^{2008})^2 - (3^{2006})^2}{(3^{2007})^2 - (3^{2005})^2}$$

simplifies to which of the following?

- (A) 1 (B) $\frac{9}{4}$ (C) 3 (D) $\frac{9}{2}$ (E) 9

- 8 Heather compares the price of a new computer at two different stores. Store A offers 15% off the sticker price followed by a \$90 rebate, and store B offers 25% off the same sticker price with no rebate. Heather saves \$15 by buying the computer at store A instead of store B. What is the sticker price of the computer, in dollars?

- (A) 750 (B) 900 (C) 1000 (D) 1050 (E) 1500

- 9 Suppose that

$$\frac{2x}{3} - \frac{x}{6}$$

is an integer. Which of the following statements must be true about x ?

- (A) It is negative. (B) It is even, but not necessarily a multiple of 3. (C) It is a multiple of 3, but not necessarily a multiple of 6. (D) It is a multiple of 6, but not necessarily a multiple of 12. (E) It is a multiple of 12.

- 10 Each of the sides of a square S_1 with area 16 is bisected, and a smaller square S_2 is constructed using the bisection points as vertices. The same process is carried out on S_2 to construct an even smaller square S_3 . What is the area of S_3 ?

- (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 3 (E) 4

- 11 While Steve and LeRoy are fishing 1 mile from shore, their boat springs a leak, and water comes in at a constant rate of 10 gallons per minute. The boat will sink if it takes in more than 30 gallons of water. Steve starts rowing toward the shore at a constant rate of 4 miles per hour while LeRoy bails water out of the boat. What is the slowest rate, in gallons per minute, at which LeRoy can bail if they are to reach the shore without sinking?

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10

- 12 In a collection of red, blue, and green marbles, there are 25% more red marbles than blue marbles, and there are 60% more green marbles than red marbles. Suppose that there are r red marbles. What is the total number of marbles in that collection?

- (A) $2.85r$ (B) $3r$ (C) $3.4r$ (D) $3.85r$ (E) $4.25r$

- 13 Doug can paint a room in 5 hours. Dave can paint the same room in 7 hours. Doug and Dave paint the room together and take a one-hour break for lunch. Let t be the total time, in hours, required for them to complete the job working together, including lunch. Which of the following equations is satisfied by t ?

- (A) $(\frac{1}{5} + \frac{1}{7})(t + 1) = 1$ (B) $(\frac{1}{5} + \frac{1}{7})t + 1 = 1$ (C) $(\frac{1}{5} + \frac{1}{7})t = 1$
 (D) $(\frac{1}{5} + \frac{1}{7})(t - 1) = 1$ (E) $(5 + 7)t = 1$

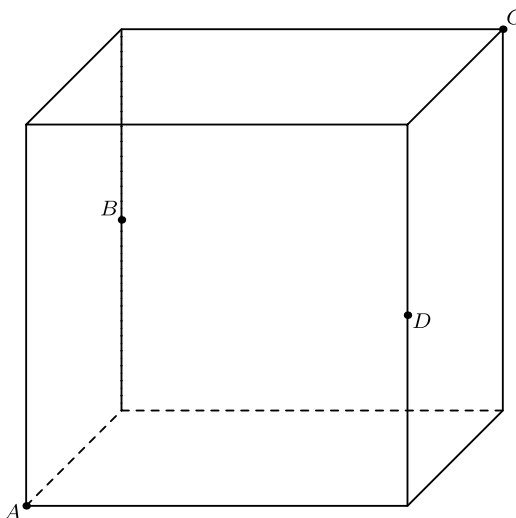
- 14 Older television screens have an aspect ratio of 4 : 3. That is, the ratio of the width to the height is 4 : 3. The aspect ratio of many movies is not 4 : 3, so they are sometimes shown on a television screen by 'letterboxing' - darkening strips of equal height at the top and bottom of the screen, as shown. Suppose a movie has an aspect ratio of 2 : 1 and is shown on an older television screen with a 27-inch diagonal. What is the height, in inches, of each darkened strip?



- (A) 2 (B) 2.25 (C) 2.5 (D) 2.7 (E) 3
-
- 15 Yesterday Han drove 1 hour longer than Ian at an average speed 5 miles per hour faster than Ian. Jan drove 2 hours longer than Ian at an average speed 10 miles per hour faster than Ian. Han drove 70 miles more than Ian. How many more miles did Jan drive than Ian?
- (A) 120 (B) 130 (C) 140 (D) 150 (E) 160
-
- 16 Points A and B lie on a circle centered at O , and $\angle AOB = 60^\circ$. A second circle is internally tangent to the first and tangent to both \overline{OA} and \overline{OB} . What is the ratio of the area of the smaller circle to that of the larger circle?
- (A) $\frac{1}{16}$ (B) $\frac{1}{9}$ (C) $\frac{1}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{4}$
-
- 17 An equilateral triangle has side length 6. What is the area of the region containing all points that are outside the triangle and not more than 3 units from a point of the triangle?
- (A) $36 + 24\sqrt{3}$ (B) $54 + 9\pi$ (C) $54 + 18\sqrt{3} + 6\pi$ (D) $(2\sqrt{3} + 3)^2 \pi$
 (E) $9(\sqrt{3} + 1)^2 \pi$
-
- 18 A right triangle has perimeter 32 and area 20. What is the length of its hypotenuse?
- (A) $\frac{57}{4}$ (B) $\frac{59}{4}$ (C) $\frac{61}{4}$ (D) $\frac{63}{4}$ (E) $\frac{65}{4}$
-
- 19 Rectangle $PQRS$ lies in a plane with $PQ = RS = 2$ and $QR = SP = 6$. The rectangle is rotated 90° clockwise about R , then rotated 90° clockwise about the point that S moved to after the first rotation. What is the length of the path traveled by point P ?
- (A) $(2\sqrt{3} + \sqrt{5})\pi$ (B) 6π (C) $(3 + \sqrt{10})\pi$ (D) $(\sqrt{3} + 2\sqrt{5})\pi$
 (E) $2\sqrt{10}\pi$
-
- 20 Trapezoid $ABCD$ has bases \overline{AB} and \overline{CD} and diagonals intersecting at K . Suppose that $AB = 9$, $DC = 12$, and the area of $\triangle AKD$ is 24. What is the area of trapezoid $ABCD$?

(A) 92 (B) 94 (C) 96 (D) 98 (E) 100

- 21 A cube with side length 1 is sliced by a plane that passes through two diagonally opposite vertices A and C and the midpoints B and D of two opposite edges not containing A and C , as shown. What is the area of quadrilateral $ABCD$?



(A) $\frac{\sqrt{6}}{2}$ (B) $\frac{5}{4}$ (C) $\sqrt{2}$ (D) $\frac{3}{2}$ (E) $\sqrt{3}$

- 22 Jacob uses the following procedure to write down a sequence of numbers. First he chooses the first term to be 6. To generate each succeeding term, he flips a fair coin. If it comes up heads, he doubles the previous term and subtracts 1. If it comes up tails, he takes half of the previous term and subtracts 1. What is the probability that the fourth term in Jacob's sequence is an integer?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$ (E) $\frac{3}{4}$

- 23 Two subsets of the set $S = \{a, b, c, d, e\}$ are to be chosen so that their union is S and their intersection contains exactly two elements. In how many ways can this be done, assuming that the order in which the subsets are chosen does not matter?

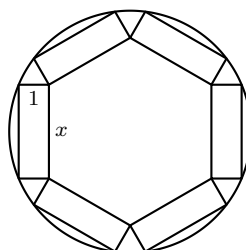
(A) 20 (B) 40 (C) 60 (D) 160 (E) 320

- 24 Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

- 25 A round table has radius 4. Six rectangular place mats are placed on the table. Each place mat has width 1 and length x as shown. They are positioned so that each mat has two corners on the edge of the table, these two corners being end points of the same side of length x . Further,

the mats are positioned so that the inner corners each touch an inner corner of an adjacent mat. What is x ?



- (A) $2\sqrt{5} - \sqrt{3}$ (B) 3 (C) $\frac{3\sqrt{7}-\sqrt{3}}{2}$ (D) $2\sqrt{3}$ (E) $\frac{5+2\sqrt{3}}{2}$

– B

– February 27th

- 1 A basketball player made 5 baskets during a game. Each basket was worth either 2 or 3 points. How many different numbers could represent the total points scored by the player?
 (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

- 2 A 4×4 block of calendar dates is shown. The order of the numbers in the second row is to be reversed. Then the order of the numbers in the fourth row is to be reversed. Finally, the numbers on each diagonal are to be added. What will be the positive difference between the two diagonal sums?

1	2	3	4
8	9	10	11
15	16	17	18
22	23	24	25

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10

- 3 Assume that x is a positive real number. Which is equivalent to $\sqrt[3]{x\sqrt{x}}$?
 (A) $x^{1/6}$ (B) $x^{1/4}$ (C) $x^{3/8}$ (D) $x^{1/2}$ (E) x

- 4 A semipro baseball league has teams with 21 players each. League rules state that a player must be paid at least \$15,000, and that the total of all players' salaries for each team cannot exceed \$700,000. What is the maximum possible salary, in dollars, for a single player?
 (A) 270,000 (B) 385,000 (C) 400,000 (D) 430,000 (E) 700,000

- 5 For real numbers a and b , define $a\$b = (a - b)^2$. What is $(x - y)^2\$(y - x)^2$?
 (A) 0 (B) $x^2 + y^2$ (C) $2x^2$ (D) $2y^2$ (E) $4xy$

- 6 Points B and C lie on \overline{AD} . The length of \overline{AB} is 4 times the length of \overline{BD} , and the length of \overline{AC} is 9 times the length of \overline{CD} . The length of \overline{BC} is what fraction of the length of \overline{AD} ?
(A) $\frac{1}{36}$ (B) $\frac{1}{13}$ (C) $\frac{1}{10}$ (D) $\frac{5}{36}$ (E) $\frac{1}{5}$
- 7 An equilateral triangle of side length 10 is completely filled in by non-overlapping equilateral triangles of side length 1. How many small triangles are required?
(A) 10 (B) 25 (C) 100 (D) 250 (E) 1000
- 8 A class collects \$50 to buy flowers for a classmate who is in the hospital. Roses cost \$3 each, and carnations cost \$2 each. No other flowers are to be used. How many different bouquets could be purchased for exactly \$50?
(A) 1 (B) 7 (C) 9 (D) 16 (E) 17
- 9 A quadratic equation $ax^2 - 2ax + b = 0$ has two real solutions. What is the average of the solutions?
(A) 1 (B) 2 (C) $\frac{b}{a}$ (D) $\frac{2b}{a}$ (E) $\sqrt{2b - a}$
- 10 Points A and B are on a circle of radius 5 and $AB = 6$. Point C is the midpoint of the minor arc AB . What is the length of the line segment AC ?
(A) $\sqrt{10}$ (B) $\frac{7}{2}$ (C) $\sqrt{14}$ (D) $\sqrt{15}$ (E) 4
- 11 Suppose that (u_n) is a sequence of real numbers satisfying $u_{n+2} = 2u_{n+1} + u_n$, and that $u_3 = 9$ and $u_6 = 128$. What is u_5 ?
(A) 40 (B) 53 (C) 68 (D) 88 (E) 104
- 12 Postman Pete has a pedometer to count his steps. The pedometer records up to 99999 steps, then flips over to 00000 on the next step. Pete plans to determine his mileage for a year. On January 1 Pete sets the pedometer to 00000. During the year, the pedometer flips from 99999 to 00000 forty-four times. On December 31 the pedometer reads 50000. Pete takes 1800 steps per mile. Which of the following is closest to the number of miles Pete walked during the year?
(A) 2500 (B) 3000 (C) 3500 (D) 4000 (E) 4500
- 13 For each positive integer n , the mean of the first n terms of a sequence is n . What is the 2008th term of the sequence?
(A) 2008 (B) 4015 (C) 4016 (D) 4,030,056 (E) 4,032,064
- 14 Triangle OAB has $O = (0, 0)$, $B = (5, 0)$, and A in the first quadrant. In addition, $\angle ABO = 90^\circ$ and $\angle AOB = 30^\circ$. Suppose that \overline{OA} is rotated 90° counterclockwise about O . What are the coordinates of the image of A ?
(A) $(-\frac{10}{3}\sqrt{3}, 5)$ (B) $(-\frac{5}{3}\sqrt{3}, 5)$ (C) $(\sqrt{3}, 5)$ (D) $(\frac{5}{3}\sqrt{3}, 5)$
(E) $(\frac{10}{3}\sqrt{3}, 5)$

-
- 15 How many right triangles have integer leg lengths a and b and a hypotenuse of length $b + 1$, where $b < 100$?
(A) 6 (B) 7 (C) 8 (D) 9 (E) 10
-
- 16 Two fair coins are to be tossed once. For each head that results, one fair die is to be rolled. What is the probability that the sum of the die rolls is odd? (Note that if no die is rolled, their sum is 0.)
(A) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{43}{72}$ (D) $\frac{5}{8}$ (E) $\frac{2}{3}$
-
- 17 A poll shows that 70% of all voters approve of the mayor's work. On three separate occasions a pollster selects a voter at random. What is the probability that on exactly one of these three occasions the voter approves of the mayor's work?
(A) 0.063 (B) 0.189 (C) 0.233 (D) 0.333 (E) 0.441
-
- 18 Bricklayer Brenda would take 9 hours to build a chimney alone, and bricklayer Brandon would take 10 hours to build it alone. When they work together they talk a lot, and their combined output is decreased by 10 bricks per hour. Working together, they build the chimney in 5 hours. How many bricks are in the chimney?
(A) 500 (B) 900 (C) 950 (D) 1000 (E) 1900
-
- 19 A cylindrical tank with radius 4 feet and height 9 feet is lying on its side. The tank is filled with water to a depth of 2 feet. What is the volume of the water, in cubic feet?
(A) $24\pi - 36\sqrt{2}$ (B) $24\pi - 24\sqrt{3}$ (C) $36\pi - 36\sqrt{3}$ (D) $36\pi - 24\sqrt{2}$
(E) $48\pi - 36\sqrt{3}$
-
- 20 The faces of a cubical die are marked with the numbers 1, 2, 2, 3, 3, and 4. The faces of a second cubical die are marked with the numbers 1, 3, 4, 5, 6, and 8. Both dice are thrown. What is the probability that the sum of the two top numbers will be 5, 7, or 9?
(A) $\frac{5}{18}$ (B) $\frac{7}{18}$ (C) $\frac{11}{18}$ (D) $\frac{3}{4}$ (E) $\frac{8}{9}$
-
- 21 Ten chairs are evenly spaced around a round table and numbered clockwise from 1 through 10. Five married couples are to sit in the chairs with men and women alternating, and no one is to sit either next to or directly across from his or her spouse. How many seating arrangements are possible?
(A) 240 (B) 360 (C) 480 (D) 540 (E) 720
-
- 22 Three red beads, two white beads, and one blue bead are placed in a line in random order. What is the probability that no two neighboring beads are the same color?
(A) $\frac{1}{12}$ (B) $\frac{1}{10}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$
-
- 23 A rectangular floor measures a by b feet, where a and b are positive integers with $b > a$. An

artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half of the area of the entire floor. How many possibilities are there for the ordered pair (a, b) ?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

24 Quadrilateral $ABCD$ has $AB = BC = CD$, $\angle ABC = 70^\circ$, and $\angle BCD = 170^\circ$. What is the degree measure of $\angle BAD$?

(A) 75 (B) 80 (C) 85 (D) 90 (E) 95

25 Michael walks at the rate of 5 feet per second on a long straight path. Trash pails are located every 200 feet along the path. A garbage truck travels at 10 feet per second in the same direction as Michael and stops for 30 seconds at each pail. As Michael passes a pail, he notices the truck ahead of him just leaving the next pail. How many times will Michael and the truck meet?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8



— These problems are copyright © Mathematical Association of America (<http://maa.org>).
