AMC 10 2014

www.artofproblemsolving.com/community/c4812

by AlcumusGuy, El_Ectric, djmathman, bestwillcui1, alex31415, niraekjs, brandbest1, Royalreter1, bobthesmartypants, sunny2000, flamefoxx99, professordad, Quadratic64, mathman523, rrusczyk

–	A
–	February 4th
1	What is $10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1}$? **(A)** 3 **(B)** 8 **(C)** $\frac{25}{2}$ **(D)** $\frac{170}{3}$ **(E)** 170
2	Roy's cat eats $\frac{1}{3}$ of a can of cat food every morning and $\frac{1}{4}$ of a can of cat food every evening. Before feeding his cat on Monday morning, Roy opened a box containing 6 cans of cat food. On what day of the week did the cat finish eating all the cat food in the box? **(A)** Tuesday **(B)** Wednesday **(C)** Thursday **(D)** Friday **(E)** Saturday
3	Bridget bakes 48 loaves of bread for her bakery. She sells half of them in the morning for $\$2.50$ each. In the afternoon she sells two thirds of what she has left, and because they are not fresh, she charges only half price. In the late afternoon she sells the remaining loaves at a dollar each. Each loaf costs $\$0.75$ for her to make. In dollars, what is her profit for the day? **(A)** 24 **(B)** 36 **(C)** 44 **(D)** 48 **(E)** 52
4	Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible? **(A)** 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6
5	On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and median score of the students' scores on this quiz? **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5
6	Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days? **(A)** $\frac{bde}{ac}$ **(B)** $\frac{ac}{bde}$ **(C)** $\frac{abde}{c}$ **(D)** $\frac{bcde}{a}$ **(E)** $\frac{abc}{de}$

7 Nonzero real numbers x, y, a, and b satisfy $x < a$ and $y < b$. How many of the following inequalities must be true?

(I) $x + y < a + b$
(II) $x - y < a - b$
(III) $xy < ab$
(IV) $\frac{x}{y} < \frac{a}{b}$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

---

8 Which of the following numbers is a perfect square?

(A) $\frac{14!15!}{2}$ **(B)** $\frac{15!16!}{2}$ **(C)** $\frac{16!17!}{2}$ **(D)** $\frac{17!18!}{2}$ **(E)** $\frac{18!19!}{2}$

---

9 The two legs of a right triangle, which are altitudes, have lengths $2\sqrt{3}$ and 6. How long is the third altitude of the triangle?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

---

10 Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b?

(A) $a + 3$ **(B)** $a + 4$ **(C)** $a + 5$ **(D)** $a + 6$ **(E)** $a + 7$

---

11 A customer who intends to purchase an appliance has three coupons, only one of which may be used:

Coupon 1: 10% off the listed price if the listed price is at least $\$50$

Coupon 2: $\$20$ off the listed price if the listed price is at least $\$100$

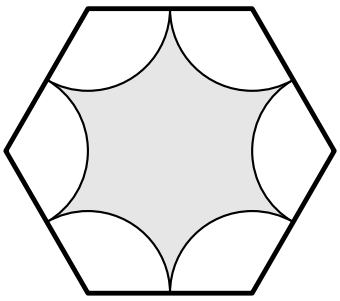
Coupon 3: 18% off the amount by which the listed price exceeds $\$100$

For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

(A) $\$179.95$ **(B)** $\$199.95$ **(C)** $\$219.95$ **(D)** $\$239.95$ **(E)** $\$259.95$

---

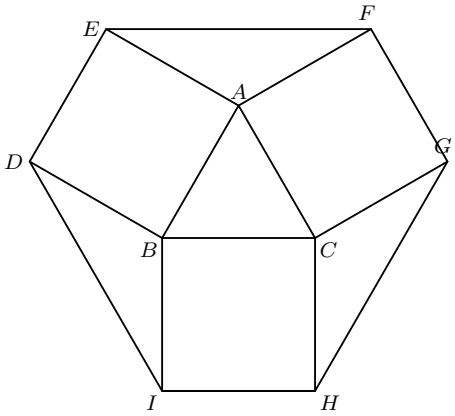
12 A regular hexagon has side length 6. Congruent arcs with radius 3 are drawn with the center at each of the vertices, creating circular sectors as shown. The region inside the hexagon but outside the sectors is shaded as shown What is the area of the shaded region?



(A) $27\sqrt{3}-9\pi$ **(B)** $27\sqrt{3}-6\pi$ **(C)** $54\sqrt{3}-18\pi$ **(D)** $54\sqrt{3}-12\pi$ **(E)** $108\sqrt{3}-9\pi$

---

13 Equilateral $\triangle ABC$ has side length 1, and squares $ABDE$, $BCHI$, $CAFG$ lie outside the triangle. What is the area of hexagon $DEFGHI$?



(A) $\frac{12+3\sqrt{3}}{4}$ **(B)** $\frac{9}{2}$ **(C)** $3+\sqrt{3}$ **(D)** $\frac{6+3\sqrt{3}}{2}$ **(E)** 6

---

14 The y-intercepts, P and Q, of two perpendicular lines intersecting at the point $A(6,8)$ have a sum of zero. What is the area of $\triangle APQ$?

(A) 45 **(B)** 48 **(C)** 54 **(D)** 60 **(E)** 72

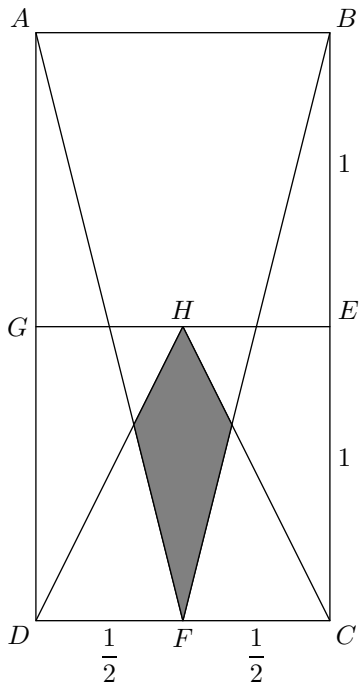
---

15 David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

(A) 140 **(B)** 175 **(C)** 210 **(D)** 245 **(E)** 280

---

16 In rectangle $ABCD$, $AB = 1$, $BC = 2$, and points E, F, and G are midpoints of $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Point H is the midpoint of $\overline{GE}$. What is the area of the shaded region?



(A) $\frac{1}{12}$ **(B)** $\frac{\sqrt{3}}{18}$ **(C)** $\frac{\sqrt{2}}{12}$ **(D)** $\frac{\sqrt{3}}{12}$ **(E)** $\frac{1}{6}$

---

17 Three fair six-sided dice are rolled. What is the probability that the values shown on two of the dice sum to the value shown on the remaining die?

(A) $\frac{1}{6}$ **(B)** $\frac{13}{72}$ **(C)** $\frac{7}{36}$ **(D)** $\frac{5}{24}$ **(E)** $\frac{2}{9}$

---

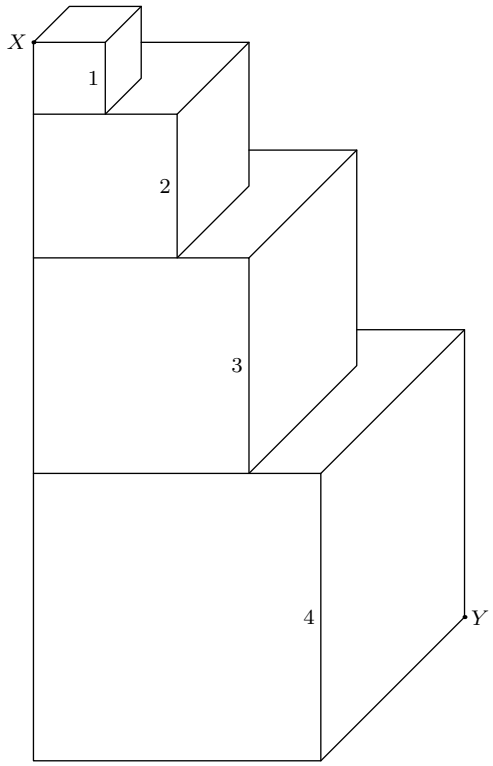
18 A square in the coordinate plane has vertices whose y-coordinates are 0, 1, 4, and 5. What is the area of the square?

(A) 16 **(B)** 17 **(C)** 25 **(D)** 26 **(E)** 27

---

19 Four cubes with edge lengths 1, 2, 3, and 4 are stacked as shown. What is the length of the portion of $\overline{XY}$ contained in the cube with edge length 3?

(A) $\frac{3\sqrt{33}}{5}$ **(B)** $2\sqrt{3}$ **(C)** $\frac{2\sqrt{33}}{3}$ **(D)** 4 **(E)** $3\sqrt{2}$



20 The product $(8)(888\ldots8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

(A) 901 **(B)** 911 **(C)** 919 **(D)** 991 **(E)** 999

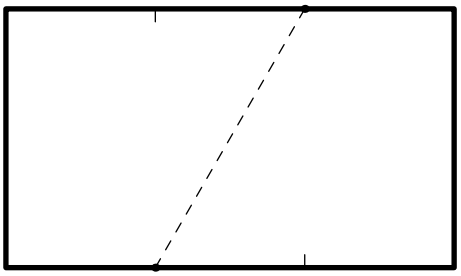
21 Positive integers a and b are such that the graphs of $y = ax+5$ and $y = 3x+b$ intersect the x-axis at the same point. What is the sum of all possible x-coordinates of these points of intersection?

(A) -20 **(B)** -18 **(C)** -15 **(D)** -12 **(E)** -8

22 In rectangle $ABCD$, $AB = 20$ and $BC = 10$. Let E be a point on $\overline{CD}$ such that $\angle CBE = 15^\circ$. What is AE?

(A) $\frac{20\sqrt{3}}{3}$ **(B)** $10\sqrt{3}$ **(C)** 18 **(D)** $11\sqrt{3}$ **(E)** 20

23 A rectangular piece of paper whose length is $\sqrt{3}$ times the width has area A. The paper is divided into equal sections along the opposite lengths, and then a dotted line is drawn from the first divider to the second divider on the opposite side as shown. The paper is then folded flat along this dotted line to create a new shape with area B. What is the ratio $B : A$?



(A) $1:2$ **(B)** $3:5$ **(C)** $2:3$ **(D)** $3:4$ **(E)** $4:5$

---

24 A sequence of natural numbers is constructed by listing the first 4, then skipping one, listing the next 5, skipping 2, listing 6, skipping 3, and, on the nth iteration, listing $n+3$ and skipping n. The sequence begins $1, 2, 3, 4, 6, 7, 8, 9, 10, 13$. What is the $500,000$th number in the sequence?

(A) $996,506$ **(B)** 996507 **(C)** 996508 **(D)** 996509 **(E)** 996510

---

25 The number 5^{867} is between 2^{2013} and 2^{2014}. How many pairs of integers (m,n) are there such that $1 \leq m \leq 2012$ and

$$5^n < 2^m < 2^{m+2} < 5^{n+1}?$$

(A) 278 **(B)** 279 **(C)** 280 **(D)** 281 **(E)** 282

– B

---

1 1. Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?

(A) 33 **(B)** 35 **(C)** 37 **(D)** 39 **(E)** 41

---

2 What is $\frac{2^3+2^3}{2^{-3}+2^{-3}}$?

(A) 16 **(B)** 24 **(C)** 32 **(D)** 48 **(E)** 64

---

3 Randy drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles, how long was Randy's trip?

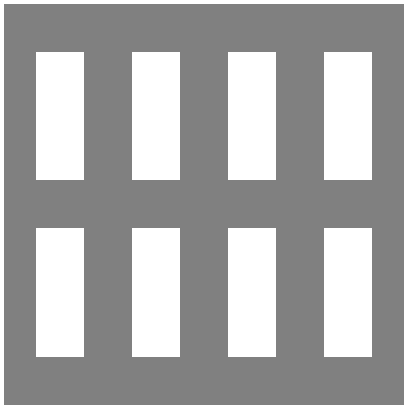
(A) 30 **(B)** $\frac{400}{11}$ **(C)** $\frac{75}{2}$ **(D)** 40 **(E)** $\frac{300}{7}$

---

4 Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana? **(A)** $\frac{3}{2}$ **(B)** $\frac{5}{3}$ **(C)** $\frac{7}{4}$ **(D)** 2 **(E)**

---

5 Doug constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is $5:2$, and the borders around and between the panes are 2 inches wide. In inches, what is the side length o the square window?



(A) 26 **(B)** 28 **(C)** 30 **(D)** 32 **(E)** 34

---

6 Orvin went to the store with just enough money to buy 30 balloons. When he arrived, he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at $\frac{1}{3}$ off the regular price. What is the greatest number of balloons Orvin could buy?

(A) 33 **(B)** 34 **(C)** 36 **(D)** 38 **(E)** 39

---

7 Suppose $A > B > 0$ and A is $x\%$ greater than B. What is x?

(A) $100\left(\frac{A-B}{B}\right)$ **(B)** $100\left(\frac{A+B}{B}\right)$ **(C)** $100\left(\frac{A+B}{A}\right)$ **(D)** $100\left(\frac{A-B}{A}\right)$ **(E)** $100\left(\frac{A}{B}\right)$

---

8 A truck travels $\frac{b}{6}$ feet every t seconds. There are 3 feet in a yard. How many yards does the truck travel in 3 minutes?

(A) $\frac{b}{1080t}$ **(B)** $\frac{30t}{b}$ **(C)** $\frac{30b}{t}$ **(D)** $\frac{10t}{b}$ **(E)** $\frac{10b}{t}$

---

9 For real numbers w and z,

$$\frac{\frac{1}{w}+\frac{1}{z}}{\frac{1}{w}-\frac{1}{z}} = 2014.$$

What is $\frac{w+z}{w-z}$?

(A) -2014 **(B)** $\frac{-1}{2014}$ **(C)** $\frac{1}{2014}$ **(D)** 1 **(E)** 2014

---

10 In the addition shown below A, B, C, and D are distinct digits. How many different values are possible for D?

$$\begin{array}{r} ABBCB \\ +\ BCADA \\ \hline DBDDD \end{array}$$

(A) 2 **(B)** 4 **(C)** 7 **(D)** 8 **(E)** 9

---

11 For the consumer, a single discount of $n\%$ is more advantageous than any of the following discounts:

(1) two successive 15% discounts (2) three successive 10% discounts (3) a 25% discount followed by a 5% discount

What is the smallest possible positive integer value of n?

(A) 27 **(B)** 28 **(C)** 29 **(D)** 31 **(E)** 33

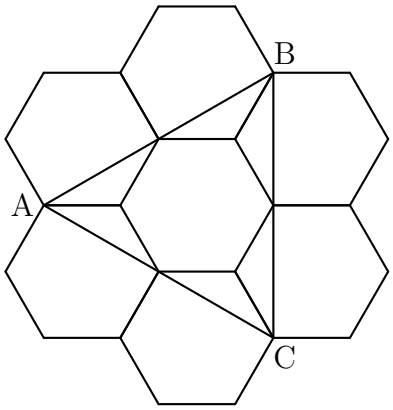
---

12 The largest divisor of $2,014,000,000$ is itself. What is its fifth largest divisor?

(A) $125,875,000$ **(B)** $201,400,000$ **(C)** $251,750,000$ **(D)** $402,800,000$ **(E)** $503,500,000$

---

13 Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?



(A) $2\sqrt{3}$ **(B)** $3\sqrt{3}$ **(C)** $1+3\sqrt{2}$ **(D)** $2+2\sqrt{3}$ **(E)** $3+2\sqrt{3}$

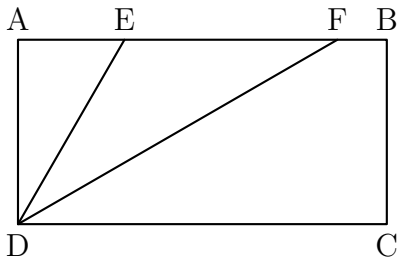
---

14 Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles were displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a+b+c \leq 7$. At the end of the trip, where the odometer showed cba miles. What is $a^2+b^2+c^2$?

(A) 26 **(B)** 27 **(C)** 36 **(D)** 37 **(E)** 41

---

15 In rectangle $ABCD$, $DC = 2CB$ and points E and F lie on $\overline{AB}$ so that $\overline{ED}$ and $\overline{FD}$ trisect $\angle ADC$ as shown. What is the ratio of the area of $\triangle DEF$ to the area of rectangle $ABCD$?



(A) $\frac{\sqrt{3}}{6}$ **(B)** $\frac{\sqrt{6}}{8}$ **(C)** $\frac{3\sqrt{3}}{16}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{\sqrt{2}}{4}$

---

16 Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?

(A) $\frac{1}{36}$ **(B)** $\frac{7}{72}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{5}{36}$ **(E)** $\frac{1}{6}$

---

17 What is the greatest power of 2 that is a factor of $10^{1002} - 4^{501}$?

(A) 2^{1002} **(B)** 2^{1003} **(C)** 2^{1004} **(D)** 2^{1005} **(E)** 2^{1006}

---

18 A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

(A) 24 **(B)** 30 **(C)** 31 **(D)** 33 **(E)** 35

---

19 Two concentric circles have radii 1 and 2. Two points on the outer circle are chosen independently and uniformly at random. What is the probability that the chord joining the two points intersects the inner circle?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

---

20 For how many integers is the number $x^4 - 51x^2 + 50$ negative?

(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

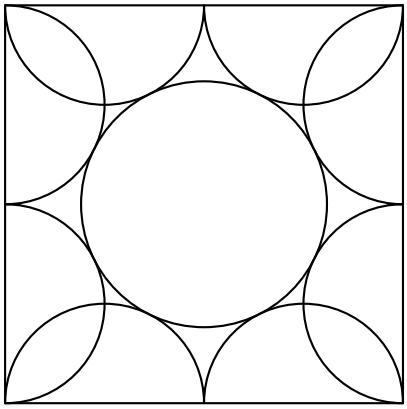
---

21 Trapezoid $ABCD$ has parallel sides $\overline{AB}$ or length 33 and $\overline{CD}$ of length 21. The other two sides are of lengths 10 and 14. The angles at A and B are acute. What is the length of the shorter diagonal of $ABCD$?

(A) $10\sqrt{6}$ **(B)** 25 **(C)** $8\sqrt{10}$ **(D)** $18\sqrt{2}$ **(E)** 26

---

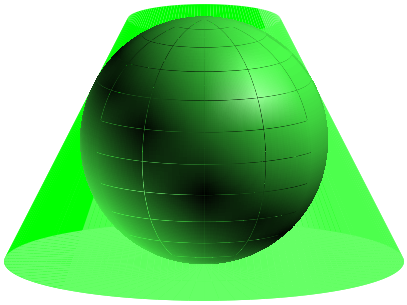
22 Eight semicircles line the inside of a square with side length 2 as shown. What is the radius of the circle tangent to all of these semicircles?



(A) $\frac{1+\sqrt{2}}{4}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{\sqrt{3}+1}{4}$ (D) $\frac{2\sqrt{3}}{5}$ (E) $\frac{\sqrt{5}}{3}$

---

23 A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?



(A) $\frac{3}{2}$ **(B)** $\frac{1+\sqrt{5}}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $\frac{3+\sqrt{5}}{2}$

---

24 The numbers 1, 2, 3, 4, 5 are to be arranged in a circle. An arrangement is *bad* if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

---

25 In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0 < N < 10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape being eaten by the snake?

(A) $\frac{32}{79}$ **(B)** $\frac{161}{384}$ **(C)** $\frac{63}{146}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

– `https://data.artofproblemsolving.com/images/maa_logo.png` These problems are copyright © Mathematical Association of America (`http://maa.org`).
