AMC 10 2015

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– A

– February 3rd

1 What is the value of $(2^0 - 1 + 5^2 + 0)^{-1} \times 5$?

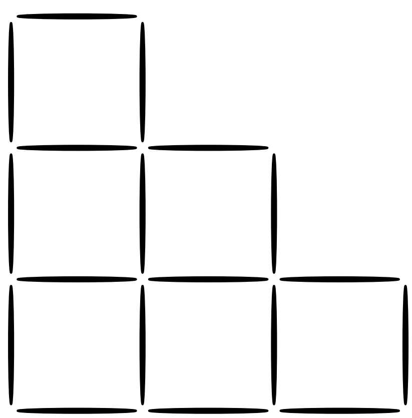
(A) -125 **(B)** -120 **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{24}$ **(E)** 25

2 A box contains a collection of triangular and square tiles. There are 25 tiles in the box, containing 84 edges total. How many square tiles are there in the box?

(A) 3 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

3 Ann made a 3-step staircase using 18 toothpicks as shown in the figure. How many toothpicks does she need to add to complete a 5-step staircase?

(A) 9 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24



4 Pablo, Sofia, and Mia got some candy eggs at a party. Pablo had three times as many eggs as Sofia, and Sofia had twice as many eggs as Mia. Pablo decides to give some of his eggs to Sofia and Mia so that all three will have the same number of eggs. What fraction of his eggs should Pablo give to Sofia?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

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5 Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the class average became 81. What was Payton's score on the test?

(A) 81 **(B)** 85 **(C)** 91 **(D)** 94 **(E)** 95

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6 The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller?

(A) $\frac{5}{4}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{9}{5}$ **(D)** 2 **(E)** $\frac{5}{2}$

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7 How many terms are there in the arithmetic sequence $13, 16, 19, \dots, 70, 73$?

(A) 20 **(B)** 21 **(C)** 24 **(D)** 60 **(E)** 61

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8 Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2:1$?

(A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

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9 Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

(A) The second height is 10% less than the first.

(B) The first height is 10% more than the second.

(C) The second height is 21% less than the first.

(D) The first height is 21% more than the second.

(E) The second height is 80% of the first.

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10 How many rearrangements of $abcd$ are there in which no two adjacent letters are also adjacent letters in the alphabet? For example, no such rearrangements could include either ab or ba.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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11 The ratio of the length to the width of a rectangle is $4:3$. If the rectangle has diagonal of length d, then the area may be expressed as kd^2 for some constant k. What is k?

(A) $\frac{2}{7}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{12}{25}$ **(D)** $\frac{16}{25}$ **(E)** $\frac{3}{4}$

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12 Points $(\sqrt{\pi}, a)$ and $(\sqrt{\pi}, b)$ are distinct points on the graph of $y^2 + x^4 = 2x^2y + 1$. What is $|a - b|$?

(A) 1 **(B)** $\frac{\pi}{2}$ **(C)** 2 **(D)** $\sqrt{1+\pi}$ **(E)** $1+\sqrt{\pi}$

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13 Claudia has 12 coins, each of which is a 5-cent coin or a 10-cent coin. There are exactly 17 different values that can be obtained as combinations of one or more of her coins. How many 10-cent coins does Claudia have?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

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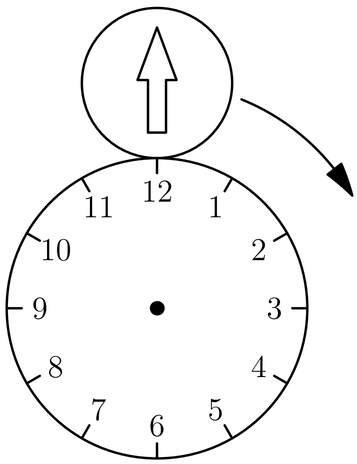
14 The diagram below shows the circular face of a clock with radius 20 cm and a circular disk with radius 10 cm externally tangent to the clock face at 12 o'clock. The disk has an arrow painted on it, initially pointing in the upward vertical direction. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent when the arrow is next pointing in the upward vertical direction?



(A) 2 o'clock **(B)** 3 o'clock **(C)** 4 o'clock **(D)** 6 o'clock **(E)** 8 o'clock

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15 Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

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16 If $y + 4 = (x-2)^2, x + 4 = (y-2)^2$, and $x \neq y$, what is the value of $x^2 + y^2$?

(A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

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17 A line that passes through the origin intersects both the line $x = 1$ and the line $y = 1 + \frac{\sqrt{3}}{3}x$. The three lines create an equilateral triangle. What is the perimeter of the triangle?

(A) $2\sqrt{6}$ **(B)** $2 + 2\sqrt{3}$ **(C)** 6 **(D)** $3 + 2\sqrt{3}$ **(E)** $6 + \frac{\sqrt{3}}{3}$

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18 Hexadecimal (base-16) numbers are written using numeric digits 0 through 9 as well as the letters A through F to represent 10 through 15. Among the first 1000 positive integers, there are n whose hexadecimal representation contains only numeric digits. What is the sum of the digits of n?

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

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19 The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. What is the area of $\triangle CDE$?

(A) $\frac{5\sqrt{2}}{3}$ **(B)** $\frac{50\sqrt{3}-75}{4}$ **(C)** $\frac{15\sqrt{3}}{8}$ **(D)** $\frac{50-25\sqrt{3}}{2}$ **(E)** $\frac{25}{6}$

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20 A rectangle has area A cm^2 and perimeter P cm, where A and P are positive integers. Which of the following numbers cannot equal $A + P$?

(A) 100 **(B)** 102 **(C)** 104 **(D)** 106 **(E)** 108

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21 Tetrahedron $ABCD$ has $AB = 5$, $AC = 3$, $BC = 4$, $BD = 4$, $AD = 3$, and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

(A) $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $\dfrac{24}{5}$ **(D)** $3\sqrt{3}$ **(E)** $\dfrac{24}{5}\sqrt{2}$

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22 Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

(A) $\dfrac{47}{256}$ **(B)** $\dfrac{3}{16}$ **(C)** $\dfrac{49}{256}$ **(D)** $\dfrac{25}{128}$ **(E)** $\dfrac{51}{256}$

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23 The zeroes of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of all possible values of a?

(A) 7 **(B)** 8 **(C)** 16 **(D)** 17 **(E)** 18

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24 For some positive integers p, there is a quadrilateral $ABCD$ with positive integer side lengths, perimeter p, right angles at B and C, $AB = 2$, and $CD = AD$. How many different values of $p < 2015$ are possible?

(A) 30 **(B)** 31 **(C)** 61 **(D)** 62 **(E)** 63

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25 Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a-b\pi}{c}$, where a, b, and c are positive integers and $\gcd(a,b,c)=1$. What is $a+b+c$?

(A) 59 **(B)** 60 **(C)** 61 **(D)** 62 **(E)** 63

– B

– February 25th

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1 What is the value of $2-(-2)^{-2}$?

(A) -2 **(B)** $\frac{1}{16}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{9}{4}$ **(E)** 6

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2 Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

(A) 3:10 PM **(B)** 3:30 PM **(C)** 4:00 PM **(D)** 4:10 PM **(E)** 4:30 PM

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3 Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

(A) 8 **(B)** 11 **(C)** 14 **(D)** 15 **(E)** 18

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4 Four siblings ordered an extra large pizza. Alex ate $\frac{1}{5}$, Beth $\frac{1}{3}$, and Cyril $\frac{1}{4}$ of the pizza. Dan got the leftovers. What is the sequence of the siblings in decreasing order of the part of pizza they consumed?

(A) Alex, Beth, Cyril, Dan **(B)** Beth, Cyril, Alex, Dan **(C)** Beth, Cyril, Dan, Alex **(D)** Beth, Dan, Cyril, Alex **(E)** Dan, Beth, Cyril, Alex

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5 David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

(A) David **(B)** Hikmet **(C)** Jack **(D)** Rand **(E)** Todd

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6 Marley practices exactly one sport each day of the week. She runs three days a week but never on two consecutive days. On Monday she plays basketball and two days later golf. She swims and plays tennis, but she never plays tennis the day after running or swimming. Which day of the week does Marley swim?

(A) Sunday **(B)** Tuesday **(C)** Thursday **(D)** Friday **(E)** Saturday

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7 Consider the operation ”minus the reciprocal of,” defined by $a\diamond b=a-\frac{1}{b}$. What is $((1\diamond 2)\diamond 3)-(1\diamond(2\diamond 3))$?

(A) $-\frac{7}{30}$ **(B)** $-\frac{1}{6}$ **(C)** 0 **(D)** $\frac{1}{6}$ **(E)** $\frac{7}{30}$

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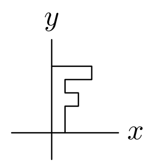
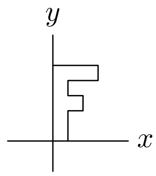
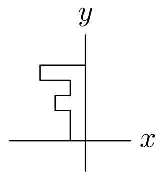
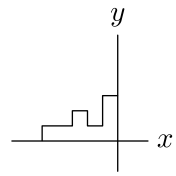
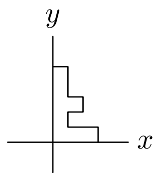
8 The letter F shown below is rotated $90°$ clockwise around the origin, then reflected in the y-axis, and then rotated a half turn around the origin. What is the final image?

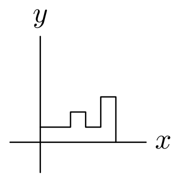


(A) 

(B) 

(C) 

(D) 

(E) 

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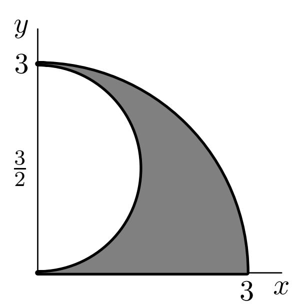
9 The shaded region below is called a shark's fin falcata, a figure studied by Leonardo da Vinci. It is bounded by the portion of the circle of radius 3 and center $(0,0)$ that lies in the first quadrant, the portion of the circle with radius $\frac{3}{2}$ and center $(0,\frac{3}{2})$ that lies in the first quadrant, and the line segment from $(0,0)$ to $(3,0)$. What is the area of the shark's fin falcata?



(A) $\frac{4\pi}{5}$ **(B)** $\frac{9\pi}{8}$ **(C)** $\frac{4\pi}{3}$ **(D)** $\frac{7\pi}{5}$ **(E)** $\frac{3\pi}{2}$

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10 What are the sign and units digit of the product of all the odd negative integers strictly greater than -2015?
(A) It is a negative number ending with a 1. **(B)** It is a positive number ending with a 1. **(C)** It is a negative
(D) It is a positive number ending with a 5. **(E)** It is a negative number ending with a 0.

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11 Among the positive integers less than 100, each of whose digits is a prime number, one is selected at random. What is the probablility that the selected number is prime?
(A) $\frac{8}{99}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{9}{20}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{9}{16}$

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12 For how many integers x is the point $(x, -x)$ inside or on the circle of radius 10 centered at $(5, 5)$?
(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

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13 The line $12x+5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?
(A) 20 **(B)** $\frac{360}{17}$ **(C)** $\frac{107}{5}$ **(D)** $\frac{43}{2}$ **(E)** $\frac{281}{13}$

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14 Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x - a)(x - b) + (x - b)(x - c) = 0$?
(A) 15 **(B)** 15.5 **(C)** 16 **(D)** 16.5 **(E)** 17

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15 The town of Hamlet has 3 people for each horse, 4 sheep for each cow, and 3 ducks for each person. Which of the following could not possibly be the total number of people, horses, sheep, cows, and ducks in Hamlet?
(A) 41 **(B)** 47 **(C)** 59 **(D)** 61 **(E)** 66

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16 Al, Bill, and Cal will each randomly be assigned a whole number from 1 to 10, inclusive, with no two of them getting the same number. What is the probability that Al's number will be a whole number multiple of Bill's and Bill's number will be a whole number multiple of Cal's?
(A) $\frac{9}{1000}$ **(B)** $\frac{1}{90}$ **(C)** $\frac{1}{80}$ **(D)** $\frac{1}{72}$ **(E)** $\frac{2}{121}$

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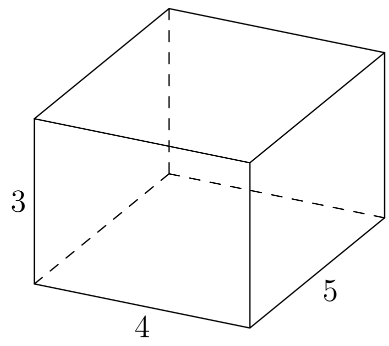
17 The centers of the faces of the right rectangular prism shown below are joined to create an octahedron, What is the volume of the octahedron?



(A) $\frac{75}{12}$ **(B)** 10 **(C)** 12 **(D)** $10\sqrt{2}$ **(E)** 15

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18 Johann has 64 fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are now heads?

(A) 32 **(B)** 40 **(C)** 48 **(D)** 56 **(E)** 64

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19 In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

(A) $12 + 9\sqrt{3}$ **(B)** $18 + 6\sqrt{3}$ **(C)** $12 + 12\sqrt{2}$ **(D)** 30 **(E)** 32

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20 Erin the ant starts at a given corner of a cube and crawls along exactly 7 edges in such a way that she visits every corner exactly once and then finds that she is unable to return along an edge to her starting point. How many paths are there meeting these conditions?

(A) 6 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 24

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21 Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose the Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

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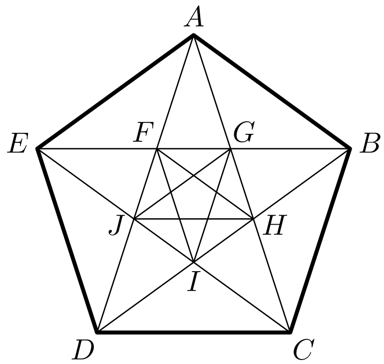
22 In the figure shown below, $ABCDE$ is a regular pentagon and $AG = 1$. What is $FG + JH + CD$?



(A) 3 **(B)** $12-4\sqrt{5}$ **(C)** $\frac{5+2\sqrt{5}}{3}$ **(D)** $1+\sqrt{5}$ **(E)** $\frac{11+11\sqrt{5}}{10}$

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23 Let n be a positive integer greater than 4 such that the decimal representation of $n!$ ends in k zeros and the decimal representation of $(2n)!$ ends in $3k$ zeros. Let s denote the sum of the four least possible values of n. What is the sum of the digits of s?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

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24 Aaron the ant walks on the coordinate plane according to the following rules. He starts at the origin $p_0=(0,0)$ facing to the east and walks one unit, arriving at $p_1=(1,0)$. For $n=1,2,3,\dots$, right after arriving at the point p_n, if Aaron can turn 90° left and walk one unit to an unvisited point p_{n+1}, he does that. Otherwise, he walks one unit straight ahead to reach p_{n+1}. Thus the sequence of points continues $p_2=(1,1), p_3=(0,1), p_4=(-1,1), p_5=(-1,0)$, and so on in a counterclockwise spiral pattern. What is p_{2015}?

(A) $(-22,-13)$ **(B)** $(-13,-22)$ **(C)** $(-13,22)$ **(D)** $(13,-22)$ **(E)** $(22,-13)$

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25 A rectangular box measures $a\times b\times c$, where a, b, and c are integers and $1\le a\le b\le c$. The volume and surface area of the box are numerically equal. How many ordered triples (a,b,c) are possible?

(A) 4 **(B)** 10 **(C)** 12 **(D)** 21 **(E)** 26

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