AMC 12/AHSME 1974

www.artofproblemsolving.com/community/c4838

by dft, ernie, Brut3Forc3, rrusczyk

1 If $x \neq 0$ or 4 and $y \neq 0$ or 6, then $\frac{2}{x} + \frac{3}{y} = \frac{1}{2}$ is equivalent to

(A) $4x + 3y = xy$ **(B)** $y = \frac{4x}{6-y}$ **(C)** $\frac{x}{2} + \frac{y}{3} = 2$
(D) $\frac{4y}{y-6} = x$ **(E)** none of these

---

2 Let x_1 and x_2 be such that $x_1 \neq x_2$ and $3x_i^2 - hx_i = b$, $i = 1, 2$. Then $x_1 + x_2$ equals

(A) $-\frac{h}{3}$ **(B)** $\frac{h}{3}$ **(C)** $\frac{b}{3}$ **(D)** $2b$ **(E)** $-\frac{b}{3}$

---

3 The coefficient of x^7 in the polynomial expansion of

$$(1 + 2x - x^2)^4$$

is

(A) -8 **(B)** 12 **(C)** 6 **(D)** -12 **(E)** none of these

---

4 What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

(A) 0 **(B)** 1 **(C)** 49 **(D)** 50 **(E)** 51

---

5 Given a quadrilateral $ABCD$ inscribed in a circle with side AB extended beyond B to point E, if $\measuredangle BAD = 92^\circ$ and $\measuredangle ADC = 68^\circ$, find $\measuredangle EBC$.

(A) 66° **(B)** 68° **(C)** 70° **(D)** 88° **(E)** 92°

---

6 For positive real numbers x and y define $x * y = \frac{x \cdot y}{x+y}$; then

(A) "*" is commutative but not associative
(B) "*" is associative but not commutative
(C) "*" is neither commutative nor associative
(D) "*" is commutative and associative
(E) none of these

---

7 A town's population increased by $1,200$ people, and then this new population decreased by 11%. The town now had 32 less people than it did before the $1,200$ increase. What is the original population?

(A) $1,200$ **(B)** $11,200$ **(C)** $9,968$ **(D)** $10,000$ **(E)** none of these

---

8 What is the smallest prime number dividing the sum $3^{11}+5^{13}$?
(A) 2 **(B)** 3 **(C)** 5 **(D)** $3^{11}+5^{13}$ **(E)** none of these

---

9 The integers greater than one are arranged in five columns as follows:

	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
	.	.	.	.

(Four consecutive integers appear in each row; in the first, third and other odd numbered rows, the integers appear in the last four columns and increase from left to right; in the second, fourth and other even numbered rows, the integers appear in the first four columns and increase from right to left.)

In which column will the number $1,000$ fall?

(A) first **(B)** second **(C)** third **(D)** fourth **(E)** fifth

---

10 What is the smallest integral value of k such that

$$2x(kx-4)-x^2+6=0$$

has no real roots?

(A) -1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

---

11 If (a,b) and (c,d) are two points on the line whose equation is $y=mx+k$, then the distance between (a,b) and (c,d), in terms of a, c, and m, is
(A) $|a-c|\sqrt{1+m^2}$ **(B)** $|a+c|\sqrt{1+m^2}$ **(C)** $\frac{|a-c|}{\sqrt{1+m^2}}$ **(D)** $|a-c|(1+m^2)$ **(E)** $|a-c|$ $|m|$

---

12 If $g(x)=1-x^2$ and $f(g(x))=\frac{1-x^2}{x^2}$ when $x\neq 0$, then $f(1/2)$ equals
(A) $3/4$ **(B)** 1 **(C)** 3 **(D)** $\sqrt{2}/2$ **(E)** $\sqrt{2}$

---

13 Which of the following is equivalent to ”If P is true, then Q is false.”?
(A) “P is true or Q is false.” **(B)** “If Q is false then P is true.” **(C)** “If P is false then Q is true.” **(D)** “If Q is true then P is false.” **(E)** “If Q is true then P is true.”

---

14 Which statement is correct?
(A) If $x<0,$ then $x^2>x.$ **(B)** If $x^2>0,$ then $x>0.$ **(C)** If $x^2>x,$ then $x>0.$ **(D)** If $x^2>x,$ then $x<0.$ **(E)** If $x<1,$ then $x^2<x.$

---

15 If $x < -2$ then $|1 - |1 + x| \, |$ equals
(A) $2 + x$ **(B)** $-2 - x$ **(C)** x **(D)** $-x$ **(E)** -2

---

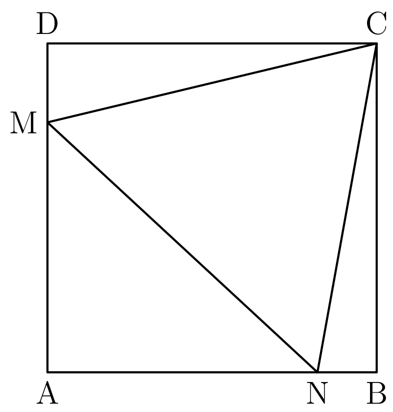
16 A circle of radius r is inscribed in a right isosceles triangle, and a circle of radius R is circumscribed about the triangle. Then R/r equals
(A) $1 + \sqrt{2}$ **(B)** $\frac{2+\sqrt{2}}{2}$ **(C)** $\frac{\sqrt{2}-1}{2}$ **(D)** $\frac{1+\sqrt{2}}{2}$ **(E)** $2(2 - \sqrt{2})$

---

17 If $i^2 = -1$, then $(1 + i)^{20} - (1 - i)^{20}$ equals
(A) -1024 **(B)** $-1024i$ **(C)** 0 **(D)** 1024 **(E)** $1024i$

---

19 In the adjoining figure $ABCD$ is a square and CMN is an equilateral triangle. If the area of $ABCD$ is one square inch, then the area of CMN in square inches is



(A) $2\sqrt{3} - 3$ **(B)** $1 - \frac{\sqrt{3}}{3}$ **(C)** $\frac{\sqrt{3}}{4}$ **(D)** $\frac{\sqrt{2}}{3}$ **(E)** $4 - 2\sqrt{3}$

---

20 Let

$$T = \frac{1}{3 - \sqrt{8}} - \frac{1}{\sqrt{8} - \sqrt{7}} + \frac{1}{\sqrt{7} - \sqrt{6}} - \frac{1}{\sqrt{6} - \sqrt{5}} + \frac{1}{\sqrt{5} - 2}$$

then
(A) $T < 1$ **(B)** $T = 1$ **(C)** $1 < T < 2$ **(D)** $T > 2$ **(E)** $T = \frac{1}{(3-\sqrt{8})(\sqrt{8}-\sqrt{7})(\sqrt{7}-\sqrt{6})(\sqrt{6}-\sqrt{5})(\sqrt{5}-2)}$

---

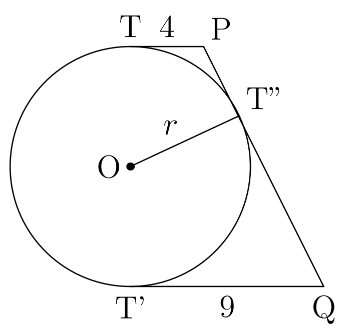
21 In a geometric series of positive terms the difference between the fifth and fourth terms is 576, and the difference between the second and first terms is 9. What is the sum of the first five terms of this series?
(A) 1061 **(B)** 1023 **(C)** 1024 **(D)** 768 **(E)** none of these

---

22 The minimum of $\sin\frac{A}{2}-\sqrt{3}\cos\frac{A}{2}$ is attained when A is
(A) -180° **(B)** 60° **(C)** 120° **(D)** 0° **(E)** none of these

---

23 In the adjoining figure TP and $T'Q$ are parallel tangents to a circle of radius r, with T and T' the points of tangency. $PT''Q$ is a third tangent with T'' as point of tangency. If $TP=4$ and $T'Q=9$ then r is



(A) $25/6$ **(B)** 6 **(C)** $25/4$
(D) a number other than $25/6, 6, 25/4$
(E) not determinable from the given information

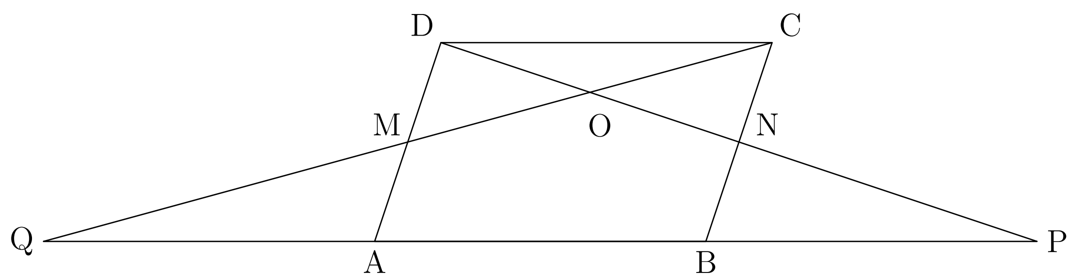
---

24 A fair die is rolled six times. The probability of rolling at least a five at least five times is

(A) $\frac{13}{729}$ **(B)** $\frac{12}{729}$ **(C)** $\frac{2}{729}$ **(D)** $\frac{3}{729}$ **(E)** none of these

---

25 In parallelogram $ABCD$ of the accompanying diagram, line DP is drawn bisecting BC at N and meeting AB (extended) at P. From vertex C, line CQ is drawn bisecting side AD at M and meeting AB (extended) at Q. Lines DP and CQ meet at O. If the area of parallelogram $ABCD$ is k, then the area of the triangle QPO is equal to



(A) k **(B)** $\frac{6k}{5}$ **(C)** $\frac{9k}{8}$ **(D)** $\frac{5k}{4}$ **(E)** $2k$

---

26 The number of distinct positive integral divisors of $(30)^4$ excluding 1 and $(30)^4$ is

(A) 100 **(B)** 125 **(C)** 123 **(D)** 30 **(E)** none of these

---

27 If $f(x) = 3x + 2$ for all real x, then the statement:
"$|f(x) + 4| < a$ whenever $|x + 2| < b$ and $a > 0$ and $b > 0$"
is true when
(A) $b \le a/3$ **(B)** $b > a/3$ **(C)** $a \le b/3$ **(D)** $a > b/3$
(E) The statement is never true.

---

28 Which of the following is satisfied by all numbers x of the form

$$x = \frac{a_1}{3} + \frac{a_2}{3^2} + \cdots + \frac{a_{25}}{3^{25}},$$

where a_1 is 0 or 2, a_2 is 0 or 2,...,a_{25} is 0 or 2?
(A) $0 \le x < 1/3$ **(B)** $1/3 \le x < 2/3$ **(C)** $2/3 \le x < 1$
(D) $0 \le x < 1/3$ or $2/3 \le x < 1$ **(E)** $1/2 \le x \le 3/4$

---

29 For $p = 1, 2, \ldots, 10$ let S_p be the sum of the first 40 terms of the arithmetic progression whose first term is p and whose common difference is $2p - 1$; then $S_1 + S_2 + \cdots + S_{10}$ is
(A) 80000 **(B)** 80200 **(C)** 80400 **(D)** 80600 **(E)** 80800

---

30 A line segment is divided so that the lesser part is to the greater part as the greater part is to the whole. If R is the ratio of the lesser part to the greater part, then the value of

$$R^{[R^{(R^2+R^{-1})}+R^{-1}]} + R^{-1}$$

is
(A) 2 **(B)** $2R$ **(C)** R^{-1} **(D)** $2 + R^{-1}$ **(E)** $2 + R$
