AMC 12/AHSME 1977

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1 If $y = 2x$ and $z = 2y$, then $x + y + z$ equals

(A) x (B) $3x$ (C) $5x$ (D) $7x$ (E) $9x$

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2 Which one of the following statements is false? All equilateral triangles are

(A) equiangular **(B)** isosceles **(C)** regular polygons **(D)** congruent to each other **(E)** similar

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3 A man has $\$2.73$ in pennies, nickels, dimes, quarters and half dollars. If he has an equal number of coins of each kind, then the total number of coins he has is

(A) 3 (B) 5 (C) 9 (D) 10 (E) 15

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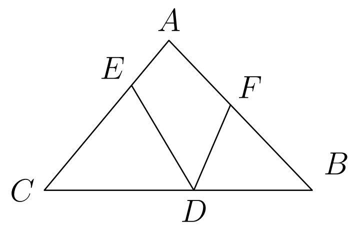

In triangle ABC, $AB = AC$ and $\measuredangle A = 80^\circ$. If points D, E, and F lie on sides BC, AC and AB, respectively, and $CE = CD$ and $BF = BD$, then $\measuredangle EDF$ equals

(A) 30° **(B)** 40° **(C)** 50° **(D)** 65° **(E)** none of these

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5 The set of all points P such that the sum of the (undirected) distances from P to two fixed points A and B equals the distance between A and B is

(A) the line segment from A to B

(B) the line passing through A and B

(C) the perpendicular bisector of the line segment from A to B

(D) an elllipse having positive area

(E) a parabola

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6 If x, y and $2x + \frac{y}{2}$ are not zero, then

$$\left(2x + \frac{y}{2}\right)\left[(2x)^{-1} + \left(\frac{y}{2}\right)^{-1}\right]$$

equals

(A) 1 **(B)** xy^{-1} **(C)** $x^{-1}y$ **(D)** $(xy)^{-1}$ **(E)** none of these

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7 If $t = \frac{1}{1-\sqrt[4]{2}}$, then t equals

(A) $(1-\sqrt[4]{2})(2-\sqrt{2})$ (B) $(1-\sqrt[4]{2})(1+\sqrt{2})$ (C) $(1+\sqrt[4]{2})(1-\sqrt{2})$

(D) $(1+\sqrt[4]{2})(1+\sqrt{2})$ (E) $-(1+\sqrt[4]{2})(1+\sqrt{2})$

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8 For every triple (a, b, c) of non-zero real numbers, form the number

$$\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}.$$

The set of all numbers formed is

(A) 0 **(B)** $\{-4, 0, 4\}$ **(C)** $\{-4, -2, 0, 2, 4\}$ **(D)** $\{-4, -2, 2, 4\}$ **(E)** none of these

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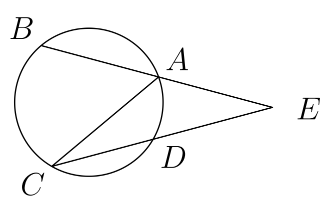

In the adjoining figure $\measuredangle E = 40^\circ$ and arc AB, arc BC, and arc CD all have equal length. Find the measure of $\measuredangle ACD$.

(A) 10° **(B)** 15° **(C)** 20° **(D)** $\left(\frac{45}{2}\right)^\circ$ **(E)** 30°

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10 If $(3x-1)^7 = a_7x^7 + a_6x^6 + \cdots + a_0$, then $a_7 + a_6 + \cdots + a_0$ equals

(A) 0 (B) 1 (C) 64 (D) -64 (E) 128

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11 For each real number x, let **[**x**]** be the largest integer not exceeding x (i.e., the integer n such that $n \le x < n+1$). Which of the following statements is (are) true?

I. **[**$x+1$**]** $=$ **[**x**]** $+1$ for all x

II. **[**$x+y$**]** $=$ **[**x**]** $+$ **[**y**]** for all x and y

III. **[**xy**]** $=$ **[**x**][**y**]** for all x and y

(A) none **(B) I** only **(C) I** and **II** only **(D) III** only **(E)** all

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13 If $a_1, a_2, a_3, \ldots$ is a sequence of positive numbers such that $a_{n+2} = a_n a_{n+1}$ for all positive integers n, then the sequence $a_1, a_2, a_3, \ldots$ is a geometric progression

(A) for all positive values of a_1 and a_2

(B) if and only if $a_1 = a_2$

(C) if and only if $a_1 = 1$

(D) if and only if $a_2 = 1$

(E) if and only if $a_1 = a_2 = 1$

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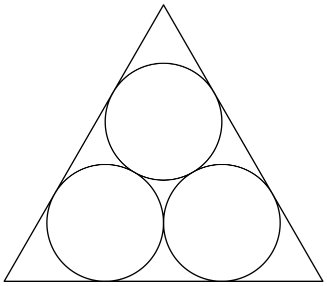
14 How many pairs (m, n) of integers satisfy the equation $m + n = mn$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** more than 4

15



Each of the three circles in the adjoining figure is externally tangent to the other two, and each side of the triangle is tangent to two of the circles. If each circle has radius three, then the perimeter of the triangle is

(A) $36+9\sqrt{2}$ **(B)** $36+6\sqrt{3}$ **(C)** $36+9\sqrt{3}$ **(D)** $18+18\sqrt{3}$ **(E)** 45

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16 If $i^2=-1$, then the sum

$$\cos 45^\circ + i\cos 135^\circ + \cdots + i^n \cos(45+90n)^\circ$$
$$+\cdots + i^{40}\cos 3645^\circ$$

equals

(A) $\frac{\sqrt{2}}{2}$ (B) $-10i\sqrt{2}$ (C) $\frac{21\sqrt{2}}{2}$
(D) $\frac{\sqrt{2}}{2}(21-20i)$ (E) $\frac{\sqrt{2}}{2}(21+20i)$

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17 Three fair dice are tossed at random (i.e., all faces have the same probability of coming up). What is the probability that the three numbers turned up can be arranged to form an arithmetic progression with common difference one?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{27}$ **(D)** $\frac{1}{54}$ **(E)** $\frac{7}{36}$

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18 If $y=(\log_2 3)(\log_3 4)\cdots(\log_n[n+1])\cdots(\log_{31} 32)$ then

(A) $4<y<5$ **(B)** $y=5$ **(C)** $5<y<6$

(D) $y=6$ **(E)** $6<y<7$

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19 Let E be the point of intersection of the diagonals of convex quadrilateral $ABCD$, and let P,Q,R, and S be the centers of the circles circumscribing triangles ABE, BCE, CDE, and ADE, respectively. Then

(A) $PQRS$ is a parallelogram

(B) $PQRS$ is a parallelogram if an only if $ABCD$ is a rhombus

(C) $PQRS$ is a parallelogram if an only if $ABCD$ is a rectangle

(D) $PQRS$ is a parallelogram if an only if $ABCD$ is a parallelogram

(E) none of the above are true

20

```
            C
          C O C
        C O N O C
      C O N T N O C
    C O N T E T N O C
  C O N T E S E T N O C
C O N T E S T S E T N O C
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For how many paths consisting of a sequence of horizontal and/or vertical line segments, with each segment connecting a pair of adjacent letters in the diagram above, is the word CONTEST spelled out as the path is traversed from beginning to end?

(A) 63 **(B)** 128 **(C)** 129 **(D)** 255 **(E)** none of these

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21 For how many values of the coefficient a do the equations

$$x^2+ax+1=0$$
$$x^2-x-a=0$$

have a common real solution?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

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22 If $f(x)$ is a real valued function of the real variable x, and $f(x)$ is not identically zero, and for all a and b

$$f(a+b)+f(a-b)=2f(a)+2f(b),$$

then for all x and y

(A) $f(0)=1$ **(B)** $f(-x)=-f(x)$

(C) $f(-x)=f(x)$ **(D)** $f(x+y)=f(x)+f(y)$

(E) there is a positive real number T such that $f(x+T)=f(x)$

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23 If the solutions of the equation $x^2+px+q=0$ are the cubes of the solutions of the equation $x^2+mx+n=0$, then

(A) $p=m^3+3mn$ **(B)** $p=m^3-3mn$

(C) $p+q=m^3$ **(D)** $\left(\frac{m}{n}\right)^2=\frac{p}{q}$ **(E)** none of these

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24 Find the sum

$$\frac{1}{1(3)}+\frac{1}{3(5)}+\cdots+\frac{1}{(2n-1)(2n+1)}+\cdots+\frac{1}{255(257)}.$$

(A) $\frac{127}{255}$ **(B)** $\frac{128}{255}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{128}{257}$ **(E)** $\frac{129}{257}$

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25 Determine the largest positive integer n such that $1005!$ is divisible by 10^n.

(A) 102 **(B)** 112 **(C)** 249 **(D)** 502 **(E)** none of these

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26 Let $a, b, c,$ and d be the lengths of sides $MN, NP, PQ,$ and QM, respectively, of quadrilateral $MNPQ$. If A is the area of $MNPQ$, then

(A) $A = \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$ if and only if $MNPQ$ is convex

(B) $A = \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$ if and only if $MNPQ$ is a rectangle

(C) $A \le \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$ if and only if $MNPQ$ is a rectangle

(D) $A \le \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$ if and only if $MNPQ$ is a parallelogram

(E) $A \ge \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$ if and only if $MNPQ$ is a parallelogram

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27 There are two spherical balls of different sizes lying in two corners of a rectangular room, each touching two walls and the floor. If there is a point on each ball which is 5 inches from each wall which that ball touches and 10 inches from the floor, then the sum of the diameters of the balls is

(A) 20 inches **(B)** 30 inches **(C)** 40 inches

(D) 60 inches **(E)** not determined by the given information

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28 Let $g(x) = x^5 + x^4 + x^3 + x^2 + x + 1$. What is the remainder when the polynomial $g(x^{12})$ is divided by the polynomial $g(x)$?

(A) 6 **(B)** $5 - x$ **(C)** $4 - x + x^2$

(D) $3 - x + x^2 - x^3$ **(E)** $2 - x + x^2 - x^3 + x^4$

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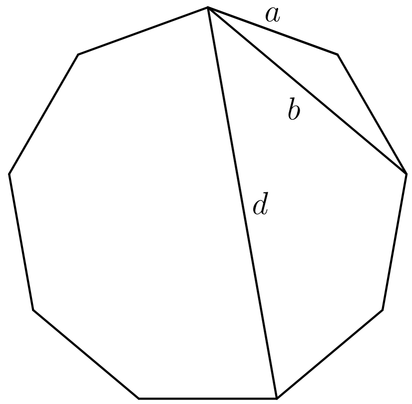
29 Find the smallest integer n such that

$$(x^2 + y^2 + z^2)^2 \le n(x^4 + y^4 + z^4)$$

for all real numbers $x, y,$ and z.

(A) 2 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** There is no such integer n.

30



If $a, b,$ and d are the lengths of a side, a shortest diagonal and a longest diagonal, respectively, of a regular nonagon (see adjoining figure), then

(A) $d = a + b$ **(B)** $d^2 = a^2 + b^2$ **(C)** $d^2 = a^2 + ab + b^2$

(D) $b = \frac{a+d}{2}$ **(E)** $b^2 = ad$

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