AMC 12/AHSME 1978

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by TheMaskedMagician, djmathman, randomusername, hshiems, rrusczyk

1 If $1-\frac{4}{x}+\frac{4}{x^2}=0$, then $\frac{2}{x}$ equals

(A) -1 **(B)** 1 **(C)** 2 **(D)** -1 or 2 **(E)** -1 or -2

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2 If four times the reciprocal of the circumference of a circle equals the diameter of the circle, then the area of the circle is

(A) $\frac{1}{\pi^2}$ **(B)** $\frac{1}{\pi}$ **(C)** 1 **(D)** π **(E)** π^2

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3 For all non-zero numbers x and y such that $x=1/y$,

$$\left(x-\frac{1}{x}\right)\left(y+\frac{1}{y}\right)$$

equals

(A) $2x^2$ **(B)** $2y^2$ **(C)** x^2+y^2 **(D)** x^2-y^2 **(E)** y^2-x^2

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4 If $a=1,\ b=10,\ c=100$, and $d=1000$, then

$$(a+b+c-d)+(a+b-c+d)+(a-b+c+d)+(-a+b+c+d)$$

is equal to

(A) 1111 **(B)** 2222 **(C)** 3333 **(D)** 1212 **(E)** 4242

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5 Four boys bought a boat for $\$60$. The first boy paid one half of the sum of the amounts paid by the other boys; the second boy paid one third of the sum of the amounts paid by the other boys; and the third boy paid one fourth of the sum of the amounts paid by the other boys. How much did the fourth boy pay?

(A) $\$10$ **(B)** $\$12$ **(C)** $\$13$ **(D)** $\$14$ **(E)** $\$15$

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6 The number of distinct pairs (x,y) of real numbers satisfying both of the following equations:

$$x=x^2+y^2,$$
$$y=2xy$$

is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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7 Opposite sides of a regular hexagon are 12 inches apart. The length of each side, in inches, is

(A) 7.5 **(B)** $6\sqrt{2}$ **(C)** $5\sqrt{2}$ **(D)** $\frac{9}{2}\sqrt{3}$ **(E)** $4\sqrt{3}$

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8 If $x \neq y$ and the sequences x, a_1, a_2, y and x, b_1, b_2, b_3, y each are in arithmetic progression, then $(a_2 - a_1)/(b_2 - b_1)$ equals

(A) $\frac{2}{3}$ **(B)** $\frac{3}{4}$ **(C)** 1 **(D)** $\frac{4}{3}$ **(E)** $\frac{3}{2}$

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9 If $x < 0$, then $\left|x - \sqrt{(x-1)^2}\right|$ equals

(A) 1 **(B)** $1 - 2x$ **(C)** $-2x - 1$ **(D)** $1 + 2x$ **(E)** $2x - 1$

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10 If B is a point on circle C with center P, then the set of all points A in the plane of circle C such that the distance between A and B is less than or equal to the distance between A and any other point on circle C is

(A) the line segment from P to B

(B) the ray beginning at P and passing through B

(C) a ray beginning at B

(D) a circle whose center is P

(E) a circle whose center is B

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11 If r is positive and the line whose equation is $x + y = r$ is tangen to the circle whose equation is $x^2 + y^2 = r$, then r equals

(A) $\frac{1}{2}$ **(B)** 1 **(C)** 2 **(D)** $\sqrt{2}$ **(E)** $2\sqrt{2}$

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12 In $\triangle ADE$, $\measuredangle ADE = 140^\circ$, points B and C lie on sides AD and AE, respectively, and points $A,\ B,\ C,\ D,\ E$ are distinct.* If lengths $AB,\ BC,\ CD,$ and DE are all equal, then the measure of $\measuredangle EAD$ is

(A) 5° **(B)** 6° **(C)** 7.5° **(D)** 8° **(E)** 10°

[size=50]* The specification that points A, B, C, D, E be distinct was not included in the original statement of the problem. If $B = D$, then $C = E$ and $\measuredangle EAD = 20^\circ$.[/size]

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13 If $a, b, c,$ and d are non-zero numbers such that c and d are the solutions of $x^2 + ax + b = 0$ and a and b are the solutions of $x^2 + cx + d = 0$, then $a + b + c + d$ equals

(A) 0 **(B)** -2 **(C)** 2 **(D)** 4 **(E)** $(-1 + \sqrt{5})/2$

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14 If an integer $n > 8$ is a solution of the equation $x^2 - ax + b = 0$ and the representation of a in the base-n number system is 18, then the base-n representation of b is

(A) 18 **(B)** 20 **(C)** 80 **(D)** 81 **(E)** 280

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15 If $\sin x + \cos x = 1/5$ and $0 \le x < \pi$, then $\tan x$ is

(A) $-\frac{4}{3}$ **(B)** $-\frac{3}{4}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{3}$

(E) not completely determined by the given information

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16 In a room containing N people, $N > 3$, at least one person has not shaken hands with everyone else in the room. What is the maximum number of people in the room that could have shaken hands with everyone else?

(A) 0 **(B)** 1 **(C)** $N-1$ **(D)** N **(E)** none of these

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17 If k is a positive number and f is a function such that, for every positive number x,

$$\left[f(x^2+1)\right]^{\sqrt{x}} = k;$$

then, for every positive number y,

$$\left[f\left(\frac{9+y^2}{y^2}\right)\right]^{\sqrt{\frac{12}{y}}}$$

is equal to

(A) $\sqrt{k}$ **(B)** $2k$ **(C)** $k\sqrt{k}$ **(D)** k^2 **(E)** $y\sqrt{k}$

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18 What is the smallest positive integer n such that $\sqrt{n} - \sqrt{n-1} < .01$?

(A) 2499 **(B)** 2500 **(C)** 2501 **(D)** $10,000$ **(E)** There is no such integer

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19 A positive integer n not exceeding 100 is chosen in such a way that if $n \le 50$, then the probability of choosing n is p, and if $n > 50$, then the probability of choosing n is $3p$. The probability that a perfect square is chosen is

(A) $.05$ **(B)** $.065$ **(C)** $.08$ **(D)** $.09$ **(E)** $.1$

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20 If a, b, c are non-zero real numbers such that

$$\frac{a+b-c}{c} = \frac{a-b+c}{b} = \frac{-a+b+c}{a},$$

and

$$x = \frac{(a+b)(b+c)(c+a)}{abc},$$

and $x < 0$, then x equals

(A) -1 **(B)** -2 **(C)** -4 **(D)** -6 **(E)** -8

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21 Consider $A = \{p, 2p, \ldots, (q-1)p\}$ and $B = \{q, 2q, \ldots, (p-1)q\}$. It's easy to see that $A \cap B = \emptyset$, because assuming for some $0 < i < q$, $0 < j < p$ we have $ip = jq$, this implies that $q|i$ and $p|j$, impossible. Also, since all their elements are less than pq, we have $(A \cup B) \in \{1, 2, \ldots, pq-1\}$. Hence we get $\prod_{k \in A \cup B} k | (pq-1)!$.
But since $A \cap B = \emptyset$, here $\prod_{k \in A \cup B} k = \prod_{k \in A} k \cdot \prod_{k \in B} k = p^{q-1}(q-1)! \cdot q^{p-1}(p-1)!$. QED.

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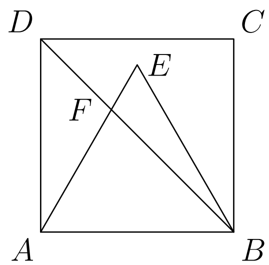
22 The following four statements, and only these are found on a card:

On this card exactly one statement is false.
On this card exactly two statements are false.
On this card exactly three statements are false.
On this card exactly four statements are false.

(Assume each statement is either true or false.) Among them the number of false statements is exactly

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

23



Vertex E of equilateral triangle ABE is in the interior of square $ABCD$, and F is the point of intersection of diagonal BD and line segment AE. If length AB is $\sqrt{1+\sqrt{3}}$ then the area of $\triangle ABF$ is

(A) 1 **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{\sqrt{3}}{2}$
(D) $4 - 2\sqrt{3}$ **(E)** $\frac{1}{2} + \frac{\sqrt{3}}{4}$

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24 If the distinct non-zero numbers $x(y-z),\ y(z-x),\ z(x-y)$ form a geometric progression with common ratio r, then r satisfies the equation

(A) $r^2+r+1=0$ **(B)** $r^2-r+1=0$ **(C)** $r^4+r^2-1=0$ **(D)** $(r+1)^4+r=0$ **(E)** $(r-1)^4+r=0$

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25 Let u be a positive number. Consider the set S of all points whose rectangular coordinates (x,y) satisfy all of the following conditions:

(i) $\frac{a}{2}\le x\le 2a$ (ii) $\frac{a}{2}\le y\le 2a$ (iii) $x+y\ge a$

(iv) $x+a\ge y$ (v) $y+a\ge x$

The boundary of set S is a polygon with

(A) 3 sides **(B)** 4 sides **(C)** 5 sides **(D)** 6 sides **(E)** 7 sides

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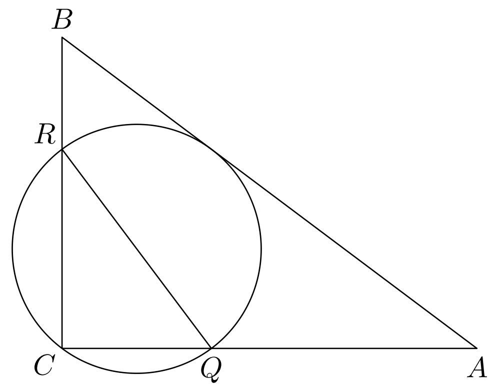
26



In $\triangle ABC$, $AB=10$ $AC=8$ and $BC=6$. Circle P is the circle with smallest radius which passes through C and is tangent to AB. Let Q and R be the points of intersection, distinct from C, of circle P with sides AC and BC, respectively. The length of segment QR is

(A) 4.75 **(B)** 4.8 **(C)** 5 **(D)** $4\sqrt{2}$ **(E)** $3\sqrt{3}$

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27 There is more than one integer greater than 1 which, when divided by any integer k such that $2\le k\le 11$, has a remainder of 1. What is the difference between the two smallest such integers?

(A) 2310 **(B)** 2311 **(C)** $27,720$ **(D)** $27,721$ **(E)** none of these

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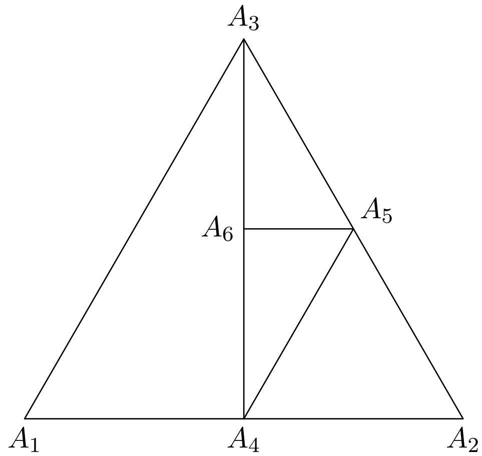
28



If $\triangle A_1A_2A_3$ is equilateral and A_{n+3} is the midpoint of line segment A_nA_{n+1} for all positive integers n, then the measure of $\measuredangle A_{44}A_{45}A_{43}$ equals

(A) 30° **(B)** 45° **(C)** 60° **(D)** 90° **(E)** 120°

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29 Sides $AB,\ BC,\ CD$ and DA, respectively, of convex quadrilateral $ABCD$ are extended past $B,\ C,\ D$ and A to points $B',\ C',\ D'$ and A'. Also, $AB = BB' = 6,\ BC = CC' = 7,\ CD = DD' = 8$ and $DA = AA' = 9$; and the area of $ABCD$ is 10. The area of $A'B'C'D'$ is

(A) 20 **(B)** 40 **(C)** 45 **(D)** 50 **(E)** 60

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30 In a tennis tournament, n women and $2n$ men play, and each player plays exactly one match with every other player. If there are no ties and the ratio of the number of matches won by women to the number of matches won by men is $7/5$, then n equals

(A) 2 **(B)** 4 **(C)** 6 **(D)** 7 **(E)** none of these
