AMC 12/AHSME 1981

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1 If $\sqrt{x+2}=2$, then $(x+2)^2$ equals
(A) $\sqrt{2}$ (B) 2 (C) 4 (D) 8 (E) 16

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2 Point E is on side AB of square $ABCD$. If EB has length one and EC has length two, then the area of the square is
(A) $\sqrt{3}$ (B) $\sqrt{5}$ (C) 3 (D) $2\sqrt{3}$ (E) 5

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3 For $x \neq 0$, $\frac{1}{x}+\frac{1}{2x}+\frac{1}{3x}$ equals
(A) $\frac{1}{2x}$ (B) $\frac{1}{6}$ (C) $\frac{5}{6x}$ (D) $\frac{11}{6x}$ (E) $\frac{1}{6x^3}$

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4 If three times the larger of two numbers is four times the smaller and the difference between the numbers is 8, the the larger of two numbers is
(A) 16 (B) 24 (C) 32 (D) 44 (E) 52

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5 In trapezoid $ABCD$, sides AB and CD are parallel, and diagonal BD and side AD have equal length. If $m\angle DBC = 110^\circ$ and $m\angle CBD = 30^\circ$, then $m\angle ADB =$
(A) 80° (B) 90° (C) 100° (D) 110° (E) 120°

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6 If $\frac{x}{x-1}=\frac{y^2+2y-1}{y^2-2y-2}$, then x equals
(A) y^2+2y-1 (B) y^2+2y-2 (C) y^2+2y+2 (D) y^2+2y+1 (E) $-y^2-2y+1$

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7 How many of the first one hundred positive integers are divisible by all of the numbers $2,3,4,5$?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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8 For all positive numbers x,y,z the product $(x+y+z)^{-1}(x^{-1}+y^{-1}+z^{-1})(xy+yz+xz)^{-1}[(xy)^{-1}+(yz)^{-1}+(xz)^{-1}]$ equals
(A) $x^{-2}y^{-2}z^{-2}$ (B) $x^{-2}+y^{-2}+z^{-2}$ (C) $(x+y+z)^{-1}$ (D) $\frac{1}{xyz}$ (E) $\frac{1}{xy+yz+xz}$

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9 In the adjoining figure, PQ is a diagonal of the cube. If PQ has length a, then the surface area of the cube is
(A) $2a^2$ (B) $2\sqrt{2}a^2$ (C) $2\sqrt{3}a^2$ (D) $3\sqrt{3}a^2$ (E) $6a^2$

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10 The lines L and K are symmetric to each other with respect to the line $y=x$. If the equation of the line L is $y=ax+b$ with $a\neq 0$ and $b \neq 0$, then the equation of K is $y=$
(A) $\frac{1}{a}x+b$ (B) $-\frac{1}{a}x+b$ (C) $\frac{1}{a}x-\frac{b}{a}$ (D) $\frac{1}{a}x+\frac{b}{a}$ (E) $\frac{1}{a}x-\frac{b}{a}$

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11 The three sides of a right triangle have integral lengths which form an arithmetic progression. One of the sides could have length
(A) 22 (B) 58 (C) 81 (D) 91 (E) 361

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12 If p, q and M are positive numbers and $q < 100$, then the number obtained by increasing M by $p\%$ and decreasing the result by $q\%$ exceeds M if and only if
(A) $p > q$ (B) $p > \frac{q}{100-q}$ (C) $p > \frac{q}{1-q}$ (D) $p > \frac{100q}{100+q}$ (E) $p > \frac{100q}{100-q}$

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13 Suppose that at the end of any year, a unit of money has lost 10% of the value it had at the beginning of that year. Find the smallest integer n such that after n years, the money will have lost at least 90% of its value. (To the nearest thousandth $\log_{10} 3 = .477$.)
(A) 14 (B) 16 (C) 18 (D) 20 (E) 22

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14 In a geometric sequence of real numbers, the sum of the first two terms is 7, and the sum of the first 6 terms is 91. The sum of the first 4 terms is
(A) 28 (B) 32 (C) 35 (D) 49 (E) 84

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15 If $b > 1$, $x > 0$ and $(2x)^{\log_b 2} - (3x)^{\log_b 3} = 0$, then x is
(A) $\frac{1}{216}$ (B) $\frac{1}{6}$ (C) 1 (D) 6 (E) not uniquely determined

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16 The base three representation of x is

$$12112211122211112222.$$

The first digit (on the left) of the base nine representation of x is
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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17 The function f is not defined for $x = 0$, but, for all non-zero real numbers x, $f(x) + 2f\left(\frac{1}{x}\right) = 3x$. The equation $f(x) = f(-x)$ is satisfied by

(A) exactly one real number (B) exactly two real numbers (C) no real numbers (D) infinitely many, but not all
(E) all non-zero real numbers

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18 The number of real solutions to the equation

$$\frac{x}{100} = \sin x$$

is

(A) 61 (B) 62 (C) 63 (D) 64 (E) 65

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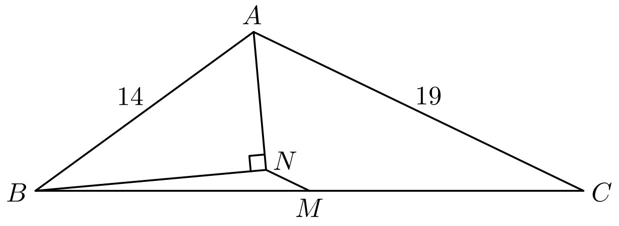
19 In $\triangle ABC$, M is the midpoint of side BC, AN bisects $\angle BAC$, $BN \perp AN$ and θ is the measure of $\angle BAC$. If sides AB and AC have lengths 14 and 19, respectively, then length MN equals



(A) 2 (B) $\frac{5}{2}$ (C) $\frac{5}{2} - \sin\theta$ (D) $\frac{5}{2} - \frac{1}{2}\sin\theta$ (E) $\frac{5}{2} - \frac{1}{2}\sin\left(\frac{1}{2}\theta\right)$

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20 A ray of light originates from point A and and travels in a plane, being reflected n times between lines AD and CD, before striking a point B (which may be on AD or CD) perpendicularly and retracing its path to A. (At each point of reflection the light makes two equal angles as indicated in the adjoining figure. The figure shows the light path for $n = 3$.) If $\measuredangle CDA = 8^\circ$, what is the largest value n can have?
(A) 6 (B) 10 (C) 38 (D) 98 (E) There is no largest value.

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21 In a triangle with sides of lengths $a, b,$ and c, $(a+b+c)(a+b-c) = 3ab$. The measure of the angle opposite the side length c is
(A) 15° (B) 30° (C) 45° (D) 60° (E) 150°

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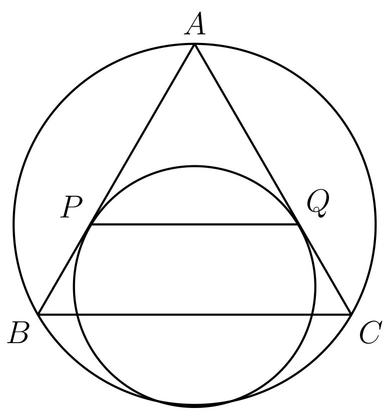
22 How many lines in a three dimensional rectangular coordiante system pass through four distinct points of the form (i, j, k) where $i, j,$ and k are positive integers not exceeding four?
(A) 60 (B) 64 (C) 72 (D) 76 (E) 100

23



Equilateral $\triangle ABC$ is inscribed in a circle. A second circle is tangent internally to the circumcircle at T and tangent to sides AB and AC at points P and Q. If side BC has length 12, then segment PQ has length
(A) 6 **(B)** $6\sqrt{3}$ **(C)** 8 **(D)** $8\sqrt{3}$ **(E)** 9

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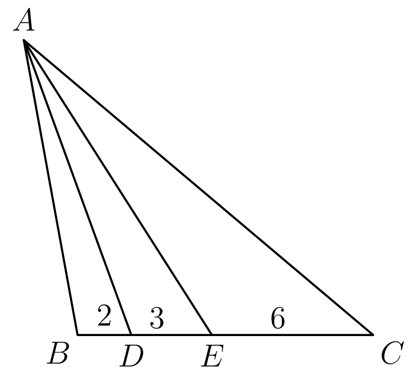
24 If θ is a constant such that $0 < \theta < \pi$ and $x + \frac{1}{x} = 2\cos\theta$. then for each positive integer n, $x^n + \frac{1}{x^n}$ equals
(A) $2\cos\theta$ **(B)** $2^n\cos\theta$ **(C)** $2\cos^n\theta$ **(D)** $2\cos n\theta$ **(E)** $2^n\cos^n\theta$

25



In triangle ABC in the adjoining figure, AD and AE trisect $\angle BAC$. The lengths of BD, DE and EC are 2, 3, and 6, respectively. The length of the shortest side of $\triangle ABC$ is
(A) $2\sqrt{10}$ **(B)** 11 **(C)** $6\sqrt{6}$ **(D)** 6 **(E)** not uniquely determined by the given information

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26 Alice, Bob, and Carol repeatedly take turns tossing a die. Alice begins; Bob always follows Alice; Carol always follows Bob; and Alice always follows Carol. Find the probability that Carol will be the first one to toss a six. (The probability of obtaining a six on any toss is $\frac{1}{6}$, independent of the outcome of any other toss.)
(A) $\frac{1}{3}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{5}{18}$ **(D)** $\frac{25}{91}$ **(E)** $\frac{36}{91}$

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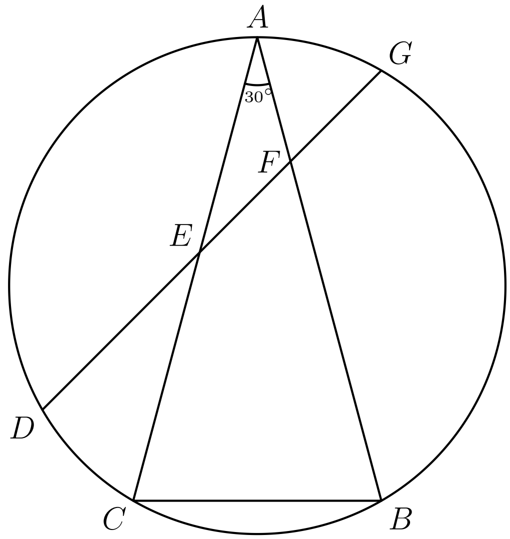
27 In the adjoining figure triangle ABC is inscribed in a circle. Point D lies on $\overset{\frown}{AC}$ with $\overset{\frown}{DC} = 30^\circ$, and point G lies on $\overset{\frown}{BA}$ with $\overset{\frown}{BG} > \overset{\frown}{GA}$. Side AB and side AC each have length equal to the length of chord DG, and $\angle CAB = 30^\circ$. Chord DG intersects sides AC and AB at E and F, respectively. The ratio of the area of $\triangle AFE$ to the area of $\triangle ABC$ is



(A) $\frac{2-\sqrt{3}}{3}$ **(B)** $\frac{2\sqrt{3}-3}{3}$ **(C)** $7\sqrt{3}-12$ **(D)** $3\sqrt{3}-5$ **(E)** $\frac{9-5\sqrt{3}}{3}$

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28 Consider the set of all equations $x^3 + a_2x^2 + a_1x + a_0 = 0$, where a_2, a_1, a_0 are real constants and $|a_i| < 2$ for $i = 0, 1, 2$. Let r be the largest positive real number which satisfies at least one of these equations. Then
(A) $1 < r < \frac{3}{2}$ **(B)** $\frac{3}{2} < r < 2$ **(C)** $2 < r < \frac{5}{2}$ **(D)** $\frac{5}{2} < r < 3$
(E) $3 < r < \frac{7}{2}$

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29 If $a > 1$, then the sum of the real solutions of

$$\sqrt{a-\sqrt{a+x}} = x$$

is equal to

(A) $\sqrt{a}-1$ **(B)** $\frac{\sqrt{a}-1}{2}$ **(C)** $\sqrt{a-1}$ **(D)** $\frac{\sqrt{a-1}}{2}$ **(E)** $\frac{\sqrt{4a-3}-1}{2}$

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30 If a, b, c, and d are the solutions of the equation $x^4 - bx - 3 = 0$, then an equation whose solutions are

$$\frac{a+b+c}{d^2}, \frac{a+b+d}{c^2}, \frac{a+c+d}{b^2}, \frac{b+c+d}{a^2}$$

is
(A) $3x^4 + bx + 1 = 0$ **(B)** $3x^4 - bx + 1 = 0$ **(C)** $3x^4 + bx^3 - 1 = 0$ **(D)** $3x^4 - bx^3 - 1 = 0$ **(E)** none of these

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