AMC 12/AHSME 1983

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1 If $x \neq 0$, $\frac{x}{2} = y^2$ and $\frac{x}{4} = 4y$, then x equals

(A) 8 **(B)** 16 **(C)** 32 **(D)** 64 **(E)** 128

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2 Point P is outside circle C on the plane. At most how many points on C are 3cm from P?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 8

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3 Three primes $p, q,$ and r satisfy $p + q = r$ and $1 < p < q$. Then p equals

(A) 2 **(B)** 3 **(C)** 7 **(D)** 13 **(E)** 17

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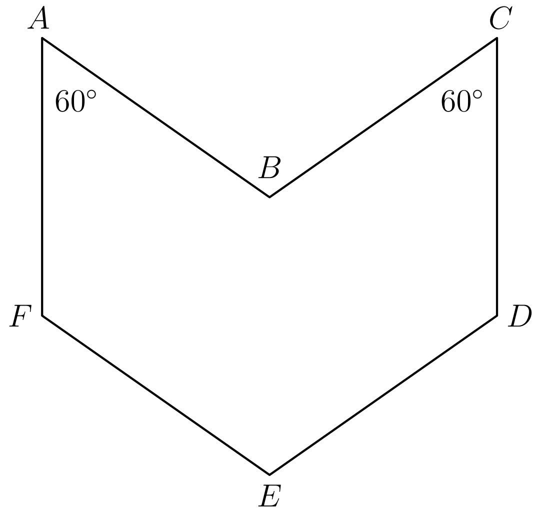
4 In the adjoining plane figure, sides AF and CD are parallel, as are sides AB and EF, and sides BC and ED. Each side has length of 1. Also, $\measuredangle FAB = \measuredangle BCD = 60^\circ$. The area of the figure is



(A) $\frac{\sqrt{3}}{2}$ **(B)** 1 **(C)** $\frac{3}{2}$ **(D)** $\sqrt{3}$ **(E)** 2

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5 Triangle ABC has a right angle at C. If $\sin A = \frac{2}{3}$, then $\tan B$ is

(A) $\frac{3}{5}$ **(B)** $\frac{\sqrt{5}}{3}$ **(C)** $\frac{2}{\sqrt{5}}$ **(D)** $\frac{\sqrt{5}}{2}$ **(E)** $\frac{5}{3}$

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6 When

$$x^5, \quad x+\frac{1}{x} \quad \text{and} \quad 1+\frac{2}{x}+\frac{3}{x^2}$$

are multiplied, the product is a polynomial of degree

(A) 2 **(B)** 3 **(C)** 6 **(D)** 7 **(E)** 8

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7 Alice sells an item at $\$10$ less than the list price and receives 10% of her selling price as her commission. Bob sells the same item at $\$20$ less than the list price and receives 20% of his selling price as his commission. If they both get the same commission, then the list price is

(A) $\$20$ **(B)** $\$30$ **(C)** $\$50$ **(D)** $\$70$ **(E)** $\$100$

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8 Let $f(x)=\frac{x+1}{x-1}$. Then for $x^2\neq 1$, $f(-x)$ is

(A) $\frac{1}{f(x)}$ **(B)** $-f(x)$ **(C)** $\frac{1}{f(-x)}$ **(D)** $-f(-x)$ **(E)** $f(x)$

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9 In a certain population the ratio of the number of women to the number of men is 11 to 10. If the average (arithmetic mean) age of the women is 34 and the average age of the men is 32, then the average age of the population is

(A) $32\frac{9}{10}$ **(B)** $32\frac{20}{21}$ **(C)** 33 **(D)** $33\frac{1}{21}$ **(E)** $33\frac{1}{10}$

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10 Segment AB is both a diameter of a circle of radius 1 and a side of an equilateral triangle ABC. The circle also intersects AC and BD at points D and E, respectively. The length of AE is

(A) $\frac{3}{2}$ (B) $\frac{5}{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$ (E) $\frac{2+\sqrt{3}}{2}$

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11 Simplify $\sin(x-y)\cos y+\cos(x-y)\sin y$.

(A) 1 **(B)** $\sin x$ **(C)** $\cos x$ **(D)** $\sin x\cos 2y$ **(E)** $\cos x\cos 2y$

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12 If $\log_7\Big(\log_3(\log_2 x)\Big)=0$, then $x^{-1/2}$ equals

(A) $\frac{1}{3}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{3\sqrt{3}}$ (D) $\frac{1}{\sqrt{42}}$ (E) none of these

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13 If $xy=a, xz=b$, and $yz=c$, and none of these quantities is zero, then $x^2+y^2+z^2$ equals:

(A) $\frac{ab+ac+bc}{abc}$ **(B)** $\frac{a^2+b^2+c^2}{abc}$ **(C)** $\frac{(a+b+c)^2}{abc}$ **(D)** $\frac{(ab+ac+bc)^2}{abc}$ **(E)** $\frac{(ab)^2+(ac)^2+(bc)^2}{abc}$

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14 The units digit of $3^{1001}7^{1002}13^{1003}$ is

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

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15 Three balls marked 1,2, and 3, are placed in an urn. One ball is drawn, its number is recorded, then the ball is returned to the urn. This process is repeated and then repeated once more, and each ball is equally likely to be drawn on each occasion. If the sum of the numbers recorded is 6, what is the probability that the ball numbered 2 was drawn all three times?
(A) $\frac{1}{27}$ (B) $\frac{1}{8}$ (C) $\frac{1}{7}$ (D) $\frac{1}{6}$ (E) $\frac{1}{3}$

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16 Let

$$x = .123456789101112\ldots998999,$$

where the digits are obtained by writing the integers 1 through 999 in order. The 1983rd digit to the right of the decimal point is

(A) 2 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 8

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17 The diagram to the right shows several numbers in the complex plane. The circle is the unit circle centered at the origin. One of these numbers is the reciprocal of F. Which one?
(A) A (B) B (C) C (D) D (E) E

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18 Let f be a polynomial function such that, for all real x,

$$f(x^2+1) = x^4+5x^2+3.$$

For all real x, $f(x^2-1)$ is

(A) x^4+5x^2+1 **(B)** x^4+x^2-3 **(C)** x^4-5x^2+1 **(D)** x^4+x^2+3 **(E)** None of these

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19 Point D is on side CB of triangle ABC. If

$$\angle CAD = \angle DAB = 60^\circ, \quad AC = 3 \quad \text{and} \quad AB = 6,$$

then the length of AD is (A) 2 (B) 2.5 (C) 3 (D) 3.5 (E) 4

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20 If $\tan\alpha$ and $\tan\beta$ are the roots of $x^2-px+q=0$, and $\cot\alpha$ and $\cot\beta$ are the roots of $x^2-rx+s=0$, then rs is necessarily (A) pq (B) $\frac{1}{pq}$ (C) $\frac{p}{q^2}$ (D) $\frac{q}{p^2}$ (E) $\frac{p}{q}$

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21 Find the smallest positive number from the numbers below (A) $10-3\sqrt{11}$ (B) $3\sqrt{11}-10$ (C) $18-5\sqrt{13}$ (D) $51-10\sqrt{26}$ (E) $10\sqrt{26}-51$

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22 Consider the two functions

$$f(x) = x^2+2bx+1 \quad \text{and} \quad g(x) = 2a(x+b),$$

where the variable x and the constants a and b are real numbers. Each such pair of the constants a and b may be considered as a point (a,b) in an $ab-$plane. Let S be the set of such points (a,b)

for which the graphs of $y = f(x)$ and $y = g(x)$ do NOT intersect (in the $xy-$ plane.). The area of S is

(A) 1 **(B)** π **(C)** 4 **(D)** 4π **(E)** infinite

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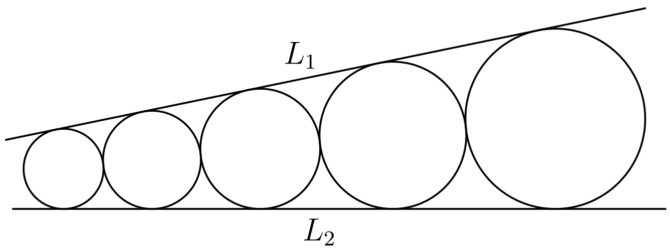
23 In the adjoining figure the five circles are tangent to one another consecutively and to the lines L_1 and L_2 (L_1 is the line that is above the circles and L_2 is the line that goes under the circles). If the radius of the largest circle is 18 and that of the smallest one is 8, then the radius of the middle circle is



(A) 12 (B) 12.5 (C) 13 (D) 13.5 (E) 14

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24 How many non-congruent right triangles are there such that the perimeter in cm and the area in cm^2 are numerically equal?
(A) none (B) 1 (C) 2 (D) 4 (E) infinitely many

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25 If $60^a = 3$ and $60^b = 5$, then $12^{[(1-a-b)/2(1-b)]}$ is

(A) $\sqrt{3}$ (B) 2 (C) $\sqrt{5}$ (D) 3 (E) $\sqrt{12}$

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26 The probability that event A occurs is $\frac{3}{4}$; the probability that event B occurs is $\frac{2}{3}$. Let p be the probability that both A and B occur. The smallest interval necessarily containing p is the interval

(A) $\left[\frac{1}{12}, \frac{1}{2}\right]$ **(B)** $\left[\frac{5}{12}, \frac{1}{2}\right]$ **(C)** $\left[\frac{1}{2}, \frac{2}{3}\right]$ **(D)** $\left[\frac{5}{12}, \frac{2}{3}\right]$ **(E)** $\left[\frac{1}{12}, \frac{2}{3}\right]$

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27 A large sphere is on a horizontal field on a sunny day. At a certain time the shadow of the sphere reaches out a distance of 10 m from the point where the sphere touches the ground. At the same instant a meter stick (held vertically with one end on the ground) casts a shadow of length 2 m. What is the radius of the sphere in meters? (Assume the sun's rays are parallel and the meter stick is a line segment.)

(A) $\frac{5}{2}$ **(B)** $9 - 4\sqrt{5}$ **(C)** $8\sqrt{10} - 23$ **(D)** $6 - \sqrt{15}$ **(E)** $10\sqrt{5} - 20$

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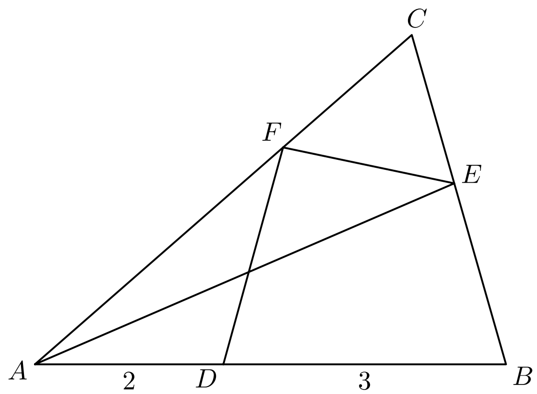
28 Triangle $\triangle ABC$ in the figure has area 10. Points D, E and F, all distinct from A, B and C, are on sides AB, BC and CA respectively, and $AD = 2$, $DB = 3$. If triangle $\triangle ABE$ and quadrilateral $DBEF$ have equal areas, then that area is



(A) 4 **(B)** 5 **(C)** 6 **(D)** $\frac{5}{3}\sqrt{10}$ **(E)** not uniquely determined

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29 A point P lies in the same plane as a given square of side 1. Let the vertices of the square, taken counterclockwise, be A, B, C and D. Also, let the distances from P to A, B and C, respectively, be u, v and w. What is the greatest distance that P can be from D if $u^2 + v^2 = w^2$?

(A) $1 + \sqrt{2}$ **(B)** $2\sqrt{2}$ **(C)** $2 + \sqrt{2}$ **(D)** $3\sqrt{2}$ **(E)** $3 + \sqrt{2}$

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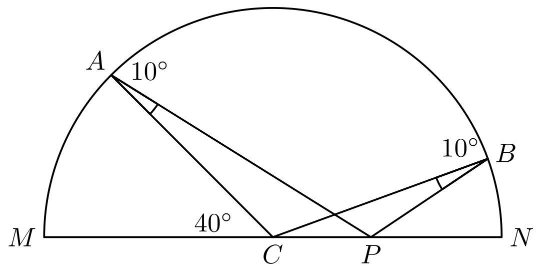
30 Distinct points A and B are on a semicircle with diameter MN and center C. The point P is on CN and $\angle CAP = \angle CBP = 10^\circ$. If $\overset{\frown}{MA} = 40^\circ$, then $\overset{\frown}{BN}$ equals



(A) 10° **(B)** 15° **(C)** 20° **(D)** 25° **(E)** 30°

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