AMC 12/AHSME 1985

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1 If $2x+1=8$, then $4x+1=$
(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

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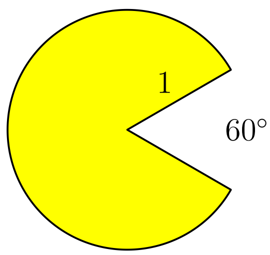
2 In an arcade game, the "monster" is the shaded sector of a circle of radius 1 cm, as shown in the figure. The missing piece (the mouth) has central angle 60°. What is the perimeter of the monster in cm?



(A) $\pi+2$ **(B)** 2π **(C)** $\frac{5}{3}\pi$ **(D)** $\frac{5}{6}\pi+2$ **(E)** $\frac{5}{3}\pi+2$

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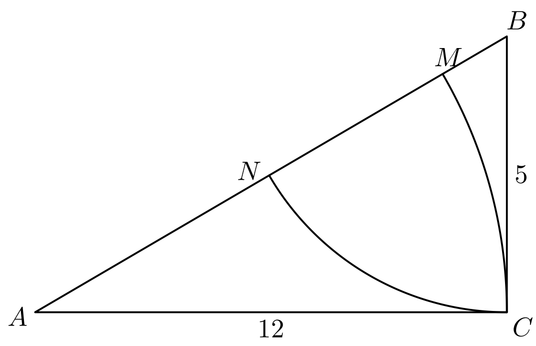
3 In right $\triangle ABC$ with legs 5 and 12, arcs of circles are drawn, one with center A and radius 12, the other with center B and radius 5. They intersect the hypotenuse at M and N. Then, MN has length:



(A) 2 **(B)** $\frac{13}{5}$ **(C)** 3 **(D)** 4 **(E)** $\frac{24}{5}$

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4 A large bag of coins contains pennies, dimes, and quarters. There are twice as many dimes as

pennies and three times as many quarters as dimes. An amount of money which could be in the bag is
(A) $\$306$ **(B)** $\$333$ **(C)** $\$342$ **(D)** $\$348$ **(E)** $\$360$

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5 Which terms must be removed from the sum

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12}$$

if the sum of the remaining terms is equal to 1?
(A) $\frac{1}{4}$ and $\frac{1}{8}$ **(B)** $\frac{1}{4}$ and $\frac{1}{12}$ **(C)** $\frac{1}{8}$ and $\frac{1}{12}$ **(D)** $\frac{1}{6}$ and $\frac{1}{10}$ **(E)** $\frac{1}{8}$ and $\frac{1}{10}$

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6 One student in a class of boys and girls is chosen to represent the class. Each student is equally likely to be chosen and the probability that a boy is chosen is $\frac{2}{3}$ of the probability that a girl is chosen. The ratio of the number of boys to the total number of boys and girls is
(A) $\frac{1}{3}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{2}{3}$

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7 In some computer languages (such as APL), when there are no parentheses in an algebraic expression, the operations are grouped from left to right. Thus, $a \times b - c$ in such languages means the same as $a(b-c)$ in ordinary algebraic notation. If $a \div b - c + d$ is evaluated in such a language, the result in ordinary algebraic notation would be
(A) $\frac{a}{b}-c+d$ **(B)** $\frac{a}{b}-c-d$ **(C)** $\frac{d+c-b}{a}$ **(D)** $\frac{a}{b-c+d}$ **(E)** $\frac{a}{b-c-d}$

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8 Let a, a', b, and b' be real numbers with a and a' nonzero. The solution to $ax+b=0$ is less than the solution to $a'x+b'=0$ if and only if
(A) $a'b<ab'$ **(B)** $ab'<a'b$ **(C)** $ab<a'b'$ **(D)** $\frac{b}{a}<\frac{b'}{a'}$ **(E)** $\frac{b'}{a'}<\frac{b}{a}$

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9 The odd positive integers $1,3,5,7,\cdots$, are arranged into in five columns continuing with the pattern shown on the right. Counting from the left, the column in which 1985 appears in is the

	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	33	35	37	39
47	45	43	41	
	49	51	53	55
•	•	•	•	
	•	•	•	•
•	•	•	•	
	•	•	•	•

(A) first **(B)** second **(C)** third **(D)** fourth **(E)** fifth

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10 An arbitrary circle can intersect the graph $y = \sin x$ in
(A) at most 2 points **(B)** at most 4 points **(C)** at most 6 points **(D)** at most 8 points
(E) more than 16 points

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11 How many **distinguishable** rearrangements of the letters in CONTEST have both the vowels first? (For instance, OETCNST is a one such arrangements but OTETSNC is not.)
(A) 60 **(B)** 120 **(C)** 240 **(D)** 720 **(E)** 2520

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12 Let's write p,q, and r as three distinct prime numbers, where 1 is not a prime. Which of the following is the smallest positive perfect cube leaving $n = pq^2r^4$ as a divisor?
(A) $p^8q^8r^8$ **(B)** $(pq^2r^2)^3$ **(C)** $(p^2q^2r^2)^3$ **(D)** $(pqr^2)^3$ **(E)** $4p^3q^3r^3$

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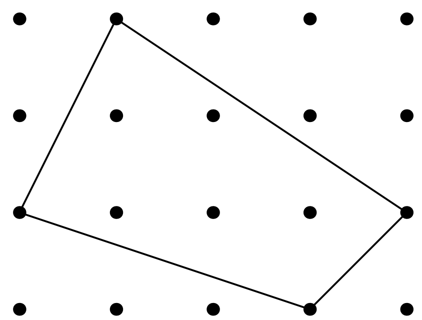
13 Pegs are put in a board 1 unit apart both horizontally and vertically. A reubber band is stretched over 4 pegs as shown in the figure, forming a quadrilateral. Its area in square units is



(A) 4 **(B)** 4.5 **(C)** 5 **(D)** 5.5 **(E)** 6

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14 Exactly three of the interior angles of a convex polygon are obtuse. What is the maximum number of sides of such a polygon?
(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

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15 If a and b are positive numbers such that $a^b = b^a$ and $b = 9a$, then the value of a is:
(A) 9 **(B)** $\frac{1}{9}$ **(C)** $\sqrt[9]{9}$ **(D)** $\sqrt[3]{9}$ **(E)** $\sqrt[4]{3}$

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16 If $A = 20^\circ$ and $B = 25^\circ$, then the value of $(1 + \tan A)(1 + \tan B)$ is
(A) $\sqrt{3}$ **(B)** 2 **(C)** $1 + \sqrt{2}$ **(D)** $2(\tan A + \tan B)$ **(E)** none of these

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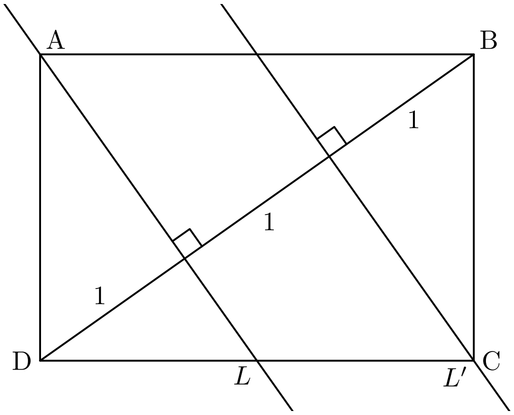
17 Diagonal DB of rectangle $ABCD$ is divided into 3 segments of length 1 by parallel lines L and L' that pass through A and C and are perpendicular to DB. The area of $ABCD$, rounded to the nearest tenth, is



(A) 4.1 **(B)** 4.2 **(C)** 4.3 **(D)** 4.4 **(E)** 4.5

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18 Six bags of marbles contain 18, 19, 21, 23, 25, and 34 marbles, respectively. One bag contains chipped marbles only. The other 5 bags contain no chipped marbles. Jane takes three of the bags and George takes two of the others. Only the bag of chipped marbles remains. If Jane gets twice as many marbles as George, how many chipped marbles are there?
(A) 18 **(B)** 19 **(C)** 21 **(D)** 23 **(E)** 25

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19 Consider the graphs $y = Ax^2$ and and $y^2 + 3 = x^2 + 4y$, where A is a positive constant and x and y are real variables. In how many points do the two graphs intersect?
(A) exactly 4 **(B)** exactly 2 **(C)** at least $1,$ but the number varies for different positive values of A
(D) 0 for at least one positive value of A **(E)** none of these

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20 A wooden cube with edge length n units (where n is an integer > 2) is painted black all over. By slices parallel to its faces, the cube is cut into n^3 smaller cubes each of unit length. If the number of smaller cubes with just one face painted black is equal to the number of smaller cubes completely free of paint, what is n?
(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** none of these

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21 How many integers x satisfy the equation

$$(x^2 - x - 1)^{x+2} = 1$$

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** none of these

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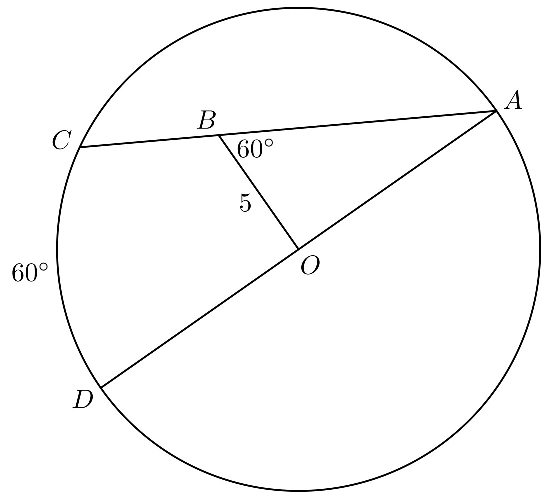
22 In a circle with center O, AD is a diameter, ABC is a chord, $BO = 5$, and $\angle ABO = \overset{\frown}{CD} = 60^\circ$. Then the length of BC is:



(A) 3 **(B)** $3+\sqrt{3}$ **(C)** $5-\frac{\sqrt{3}}{2}$ **(D)** 5 **(E)** none of the above

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23 If

$$x=\frac{-1+i\sqrt{3}}{2} \quad \text{and} \quad y=\frac{-1-i\sqrt{3}}{2},$$

where $i^2=-1$, then which of the following is *not* correct?

(A) $x^5+y^5=-1$ **(B)** $x^7+y^7=-1$ **(C)** $x^9+y^9=-1$ **(D)** $x^{11}+y^{11}=-1$ **(E)** $x^{13}+y^{13}=-1$

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24 A non-zero digit is chosen in such a way that the probability of choosing digit d is $\log_{10}(d+1)-\log_{10}d$. The probability that the digit 2 is chosen is exactly $\frac{1}{2}$ the probability that the digit chosen is in the set

(A) $\{2,3\}$ **(B)** $\{3,4\}$ **(C)** $\{4,5,6,7,8\}$ **(D)** $\{5,6,7,8,9\}$ **(E)** $\{4,5,6,7,8,9\}$

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25 The volume of a certain rectangular solid is 8 cm^3, its total surface area is 32 cm^3, and its three dimensions are in geometric progression. The sums of the lengths in cm of all the edges of this solid is

(A) 28 **(B)** 32 **(C)** 36 **(D)** 40 **(E)** 44

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26 Find the least positive integer n for which $\frac{n-13}{5n+6}$ is non-zero reducible fraction.

(A) 45 **(B)** 68 **(C)** 155 **(D)** 226 **(E)** none of these

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27 Consider a sequence $x_1,x_2,x_3,...$, defined by

$$x_1=\sqrt[3]{3}$$
$$x_2=\sqrt[3]{3}^{\sqrt[3]{3}},$$

and in general

$$x_n = (x_{n-1})^{\sqrt[3]{3}} \text{ for } n > 1.$$

What is the smallest value of n for which x_n is an integer?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 9 **(E)** 27

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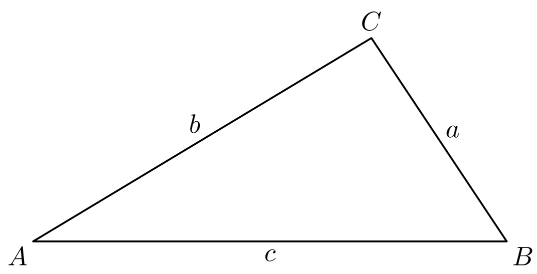
28 In $\triangle ABC$, we have $\angle C = 3\angle A$, $a = 27$, and $c = 48$. What is b?



(A) 33 **(B)** 35 **(C)** 37 **(D)** 39 **(E)** not uniquely determined

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29 In their base 10 representation, the integer a consists of a sequence of 1985 eights and the integer b consists of a sequence of 1985 fives. What is the sum of the digits of the base 10 representation of $9ab$?

(A) 15880 **(B)** 17856 **(C)** 17865 **(D)** 17874 **(E)** 19851

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30 Let $\lfloor x \rfloor$ be the greatest integer less than or equal to x. Then the number of real solutions to $4x^2 - 40\lfloor x \rfloor + 51 = 0$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4
