AMC 12/AHSME 1988

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1 $\sqrt{8}+\sqrt{18}=$

(A) $\sqrt{20}$ **(B)** $2(\sqrt{2}+\sqrt{3})$ **(C)** 7 **(D)** $5\sqrt{2}$ **(E)** $2\sqrt{13}$

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2 Triangles ABC and XYZ are similar, with A corresponding to X and B to Y. If $AB=3$, $BC=4$, and $XY=5$, then YZ is:

(A) $3\frac{3}{4}$ **(B)** 6 **(C)** $6\frac{1}{4}$ **(D)** $6\frac{2}{3}$ **(E)** 8

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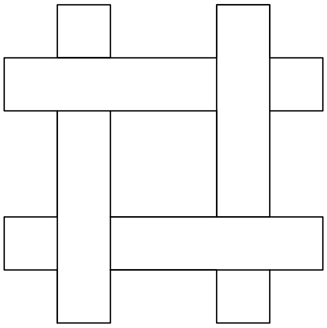
3 Four rectangular paper strips of length 10 and width 1 are put flat on a table and overlap perpendicularly as shown. How much area of the table is covered?



(A) 36 **(B)** 40 **(C)** 44 **(D)** 98 **(E)** 100

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4 The slope of the line $\frac{x}{3}+\frac{y}{2}=1$ is

(A) $-\frac{3}{2}$ **(B)** $-\frac{2}{3}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{2}$

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5 If b and c are constants and

$$(x+2)(x+b)=x^2+cx+6,$$

then c is

(A) -5 **(B)** -3 **(C)** -1 **(D)** 3 **(E)** 5

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6 A figure is an equiangular parallelogram if and only if it is a

(A) rectangle **(B)** regular polygon **(C)** rhombus **(D)** square **(E)** trapezoid

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7 Estimate the time it takes to send 60 blocks of data over a communications channel if each block consists of 512 "chunks" and the channel can transmit 120 chunks per second.

(A) 0.04 seconds **(B)** 0.4 seconds **(C)** 4 seconds **(D)** 4 minutes **(E)** 4 hours

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8 If $\frac{b}{a} = 2$ and $\frac{c}{b} = 3$, what is the ratio of $a + b$ to $b + c$?

(A) $\frac{1}{3}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

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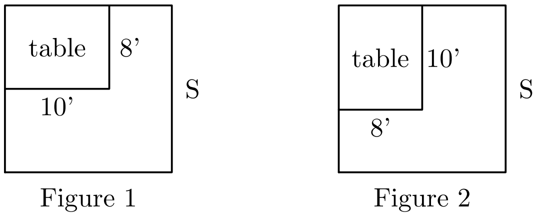
9 An $8'$ X $10'$ table sits in the corner of a square room, as in Figure 1 below. The owners desire to move the table to the position shown in Figure 2. The side of the room is S feet. What is the smallest integer value of S for which the table can be moved as desired without tilting it or taking it apart?



(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

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10 In an experiment, a scientific constant C is determined to be 2.43865 with an error of at most ± 0.00312. The experimenter wishes to announce a value for C in which every digit is significant. That is, whatever C is, the announced value must be the correct result when C is rounded to that number of digits. The most accurate value the experimenter can announce for C is

(A) 2 **(B)** 2.4 **(C)** 2.43 **(D)** 2.44 **(E)** 2.439

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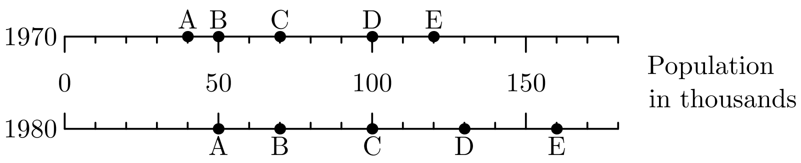
11 On each horizontal line in the figure below, the five large dots indicate the populations of cities A, B, C, D and E in the year indicated. Which city had the greatest percentage increase in population from 1970 to 1980?



(A) A **(B)** B **(C)** C **(D)** D **(E)** E

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12 Each integer 1 through 9 is written on a separate slip of paper and all nine slips are put into a hat. Jack picks one of these slips at random and puts it back. Then Jill picks a slip at random. Which digit is most likely to be the units digit of the **sum** of Jack's integer and Jill's integer?

(A) 0 **(B)** 1 **(C)** 8 **(D)** 9 **(E)** each digit is equally likely

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13 If $\sin\,x = 3\,\cos\,x$ then what is $\sin\,x\,\cos\,x$?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{3}{10}$

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14 For any real number a and positive integer k, define

$$\binom{a}{k} = \frac{a(a-1)(a-2)\cdots(a-(k-1))}{k(k-1)(k-2)\cdots(2)(1)}.$$

What is

$$\binom{-\frac{1}{2}}{100} \div \binom{\frac{1}{2}}{100}?$$

(A) -199 **(B)** -197 **(C)** -1 **(D)** 197 **(E)** 199

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15 If a and b are integers such that $x^2 - x - 1$ is a factor of $ax^3 + bx^2 + 1$, then b is

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

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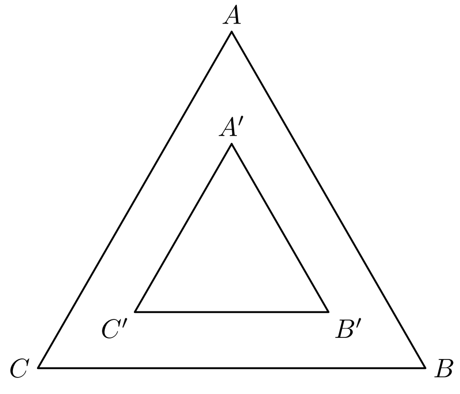
16 ABC and $A'B'C'$ are equilateral triangles with parallel sides and the same center, as in the figure. The distance between side BC and side $B'C'$ is $\frac{1}{6}$ the altitude of $\triangle ABC$. The ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$ is



(A) $\frac{1}{36}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{\sqrt{3}}{4}$ **(E)** $\frac{9+8\sqrt{3}}{36}$

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17 If $|x| + x + y = 10$ and $x + |y| - y = 12$, find $x + y$.

(A) -2 **(B)** 2 **(C)** $\frac{18}{5}$ **(D)** $\frac{22}{3}$ **(E)** 22

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18 At the end of a professional bowling tournament, the top 5 bowlers have a playoff. First#5 bowls#4. The loser receives 5th prize and the winner bowls#3 in another game. The loser of this game receives 4th prize and the winner bowls#2. The loser of this game receives 3rd prize and the winner bowls#1. The winner of this game gets 1st prize and the loser gets 2nd prize. In how many orders can bowlers#1 through#5 receive the prizes?

(A) 10 **(B)** 16 **(C)** 24 **(D)** 120 **(E)** none of these

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19 Simplify

$$\frac{bx(a^2x^2 + 2a^2y^2 + b^2y^2) + ay(a^2x^2 + 2b^2x^2 + b^2y^2)}{bx + ay}.$$

(A) $a^2x^2+b^2y^2$ **(B)** $(ax+by)^2$ **(C)** $(ax+by)(bx+ay)$ **(D)** $2(a^2x^2+b^2y^2)$ **(E)** $(bx+ay)^2$

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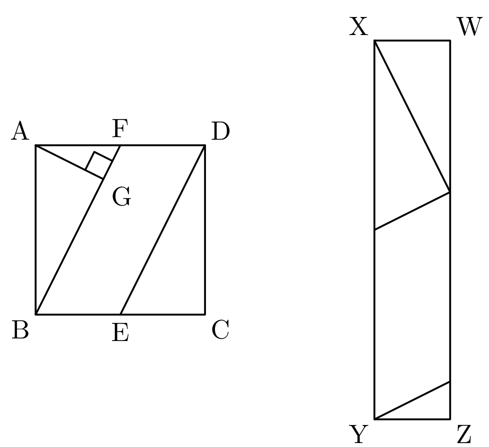
20 In one of the adjoining figures a square of side 2 is dissected into four pieces so that E and F are the midpoints of opposite sides and AG is perpendicular to BF. These four pieces can then be reassembled into a rectangle as shown in the second figure. The ratio of height to base, XY / YZ, in this rectangle is



(A) 4 **(B)** $1 + 2\sqrt{3}$ **(C)** $2\sqrt{5}$ **(D)** $\frac{8+4\sqrt{3}}{3}$ **(E)** 5

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21 The complex number z satisfies $z + |z| = 2 + 8i$. What is $|z|^2$? Note: if $z = a + bi$, then $|z| = \sqrt{a^2 + b^2}$.

(A) 68 **(B)** 100 **(C)** 169 **(D)** 208 **(E)** 289

22 For how many integers x does a triangle with side lengths $10, 24$ and x have all its angles acute?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** more than 7

23 The six edges of a tetrahedron $ABCD$ measure $7, 13, 18, 27, 36$ and 41 units. If the length of edge AB is 41, then the length of edge CD is

(A) 7 **(B)** 13 **(C)** 18 **(D)** 27 **(E)** 36

24 An isosceles trapezoid is circumscribed around a circle. The longer base of the trapezoid is 16, and one of the base angles is $\arcsin(.8)$. Find the area of the trapezoid.

(A) 72 **(B)** 75 **(C)** 80 **(D)** 90 **(E)** not uniquely determined

25 X, Y and Z are pairwise disjoint sets of people. The average ages of people in the sets X, Y, Z, $X \cup Y$, $X \cup Z$ and $Y \cup Z$ are given in the table below.

Set	X	Y	Z	$X \cup Y$	$X \cup Z$	$Y \cup Z$
Average age of people in the set	37	23	41	29	39.5	33

Find the average age of the people in set $X \cup Y \cup Z$.

(A) 33 **(B)** 33.5 **(C)** $33.6\overline{6}$ **(D)** $33.83\overline{3}$ **(E)** 34

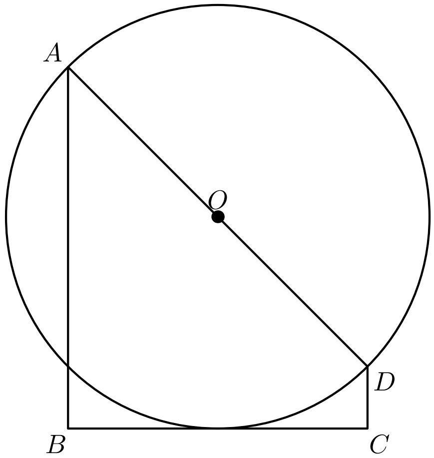
26 Suppose that p and q are positive numbers for which

$$\log_9(p) = \log_{12}(q) = \log_{16}(p+q)$$

What is the value of $\frac{q}{p}$?

(A) $\frac{4}{3}$ **(B)** $\frac{1+\sqrt{3}}{2}$ **(C)** $\frac{8}{5}$ **(D)** $\frac{1+\sqrt{5}}{2}$ **(E)** $\frac{16}{9}$

27 In the figure, $AB \perp BC$, $BC \perp CD$, and BC is tangent to the circle with center O and diameter AD. In which one of the following cases is the area of $ABCD$ an integer?



(A) $AB = 3, CD = 1$ **(B)** $AB = 5, CD = 2$ **(C)** $AB = 7, CD = 3$ **(D)** $AB = 9, CD = 4$ **(E)** $AB = 11, CD = 5$

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28 An unfair coin has probability p of coming up heads on a single toss. Let w be the probability that, in 5 independent toss of this coin, heads come up exactly 3 times. If $w = 144/625$, then

(A) p must be $2/5$ **(B)** p must be $3/5$ **(C)** p must be greater than $3/5$ **(D)** p is not uniquely determined **(E)** there is no value of p for which $w = 144/625$

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29 You plot weight (y) against height (x) for three of your friends and obtain the points (x_1, y_1), (x_2, y_2), (x_3, y_3). If

$$x_1 < x_2 < x_3 \quad \text{and} \quad x_3 - x_2 = x_2 - x_1,$$

which of the following is necessarily the slope of the line which best fits the data? "Best fits" means that the sum of the squares of the vertical distances from the data points to the line is smaller than for any other line.

(A) $\frac{y_3-y_1}{x_3-x_1}$ **(B)** $\frac{(y_2-y_1)-(y_3-y_2)}{x_3-x_1}$ **(C)** $\frac{2y_3-y_1-y_2}{2x_3-x_1-x_2}$ **(D)** $\frac{y_2-y_1}{x_2-x_1} + \frac{y_3-y_2}{x_3-x_2}$ **(E)** none of these

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30 Let $f(x) = 4x - x^2$. Give x_0, consider the sequence defined by $x_n = f(x_{n-1})$ for all $n \geq 1$. For how many real numbers x_0 will the sequence $x_0, x_1, x_2, \ldots$ take on only a finite number of different values?

(A) 0 **(B)** 1 or 2 **(C)** 3, 4, 5 or 6 **(D)** more than 6 but finitely many **(E)** infinitely many

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