**AMC 12/AHSME 2002**

www.artofproblemsolving.com/community/c4866

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| | |
|---|---|
| – | A |
| – | December 2nd |
| 1 | Compute the sum of all the roots of $(2x+3)(x-4)+(2x+3)(x-6)=0$.<br>**(A)** $7/2$ **(B)** $4$ **(C)** $5$ **(D)** $7$ **(E)** $13$ |
| 2 | Cindy was asked by her teacher to subtract $3$ from a certain number and then divide the result by $9$. Instead, she subtracted $9$ and then divided the result by $3$, giving an answer of $43$. What would her answer have been had she worked the problem correctly?<br>**(A)** $15$ **(B)** $34$ **(C)** $43$ **(D)** $51$ **(E)** $138$ |

**3** According to the standard convention for exponentiation,

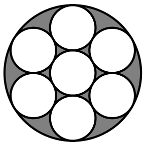
$$2^{2^{2^{2}}} = 2^{\left(2^{\left(2^2\right)}\right)} = 2^{16} = 65{,}536.$$

If the order in which the exponentiations are performed is changed, how many <u>other</u> values are possible?

**(A)** $0$ **(B)** $1$ **(C)** $2$ **(D)** $3$ **(E)** $4$

**4** Find the degree measure of an angle whose complement is $25\%$ of its supplement.
**(A)** $48$ **(B)** $60$ **(C)** $75$ **(D)** $120$ **(E)** $150$

**5** Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



**(A)** $\pi$ **(B)** $1.5\pi$ **(C)** $2\pi$ **(D)** $3\pi$ **(E)** $3.5\pi$

**6** For how many positive integers $m$ does there exist at least one positive integer $n$ such that $m \cdot n \leq m+n$?
**(A)** $4$ **(B)** $6$ **(C)** $9$ **(D)** $12$ **(E)** infinitely many

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**7** If an arc of $45^\circ$ on circle $A$ has the same length as an arc of $30^\circ$ on circle $B$, then the ratio of the area of circle $A$ to the area of circle $B$ is
**(A)** $\frac{4}{9}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{9}{4}$

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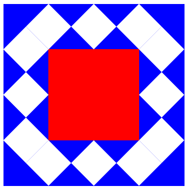
**8** Betsy designed a flag using blue triangles, small white squares, and a red center square, as shown. Let $B$ be the total area of the blue triangles, $W$ the total area of the white squares, and $R$ the area of the red square. Which of the following is correct?



**(A)** $B=W$ **(B)** $W=R$ **(C)** $B=R$ **(D)** $3B=2R$ **(E)** $2R=W$

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**9** Jamal wants to store $30$ computer files on floppy disks, each of which has a capacity of $1.44$ megabytes (MB). Three of his files require $0.8$ MB of memory each, $12$ more require $0.7$ MB each, and the remaining $15$ require $0.4$ MB each. No file can be split between floppy disks. What is the minimal number of floppy disks that will hold all the files?
**(A)** $12$ **(B)** $13$ **(C)** $14$ **(D)** $15$ **(E)** $16$

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**10** Sarah pours four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then transfers half the coffee from the first cup to the second and, after stirring thoroughly, transfers half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?
**(A)** $1/4$ **(B)** $1/3$ **(C)** $3/8$ **(D)** $2/5$ **(E)** $1/2$

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**11** Mr. Earl E. Bird leaves his house for work at exactly 8:00 A.M. every morning. When he averages $40$ miles per hour, he arrives at his workplace three minutes late. When he averages $60$ miles per hour, he arrives three minutes early. At what average speed, in miles per hour, should Mr. Bird drive to arrive at his workplace precisely on time?
**(A)** $45$ **(B)** $48$ **(C)** $50$ **(D)** $55$ **(E)** $58$

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**12** Both roots of the quadratic equation $x^2-63x+k=0$ are prime numbers. The number of possible

values of $k$ is
**(A)** $0$ **(B)** $1$ **(C)** $2$ **(D)** $3$ **(E) more than four**

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**13** Two different positive numbers $a$ and $b$ each differ from their reciprocals by 1. What is $a+b$?

**(A)** $1$ **(B)** $2$ **(C)** $\sqrt{5}$ **(D)** $\sqrt{6}$ **(E)** $3$

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**14** For all positive integers $n$, let $f(n) = \log_{2002} n^2$. Let

$$N = f(11) + f(13) + f(14)$$

Which of the following relations is true?
**(A)** $N < 1$ **(B)** $N = 1$ **(C)** $1 < N < 2$ **(D)** $N = 2$ **(E)** $N > 2$

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**15** The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is
**(A)** $11$ **(B)** $12$ **(C)** $13$ **(D)** $14$ **(E)** $15$

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**16** Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$ and Sergio randomly selects a number from the set $\{1, 2, ..., 10\}$. The probability that Sergio's number is larger than the sum of the two numbers chosen by Tina is
**(A)** $2/5$ **(B)** $9/20$ **(C)** $1/2$ **(D)** $11/20$ **(E)** $24/25$

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**17** Several sets of prime numbers, such as $\{7, 83, 421, 659\}$ use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?
**(A)** $193$ **(B)** $207$ **(C)** $225$ **(D)** $252$ **(E)** $447$

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**18** Let $C_1$ and $C_2$ be circles defined by

$$(x-10)^2 + y^2 = 36$$

and

$$(x+15)^2 + y^2 = 81,$$

respectively. What is the length of the shortest line segment $\overline{PQ}$ that is tangent to $C_1$ at $P$ and to $C_2$ at $Q$?
**(A)** $15$ **(B)** $18$ **(C)** $20$ **(D)** $21$ **(E)** $24$

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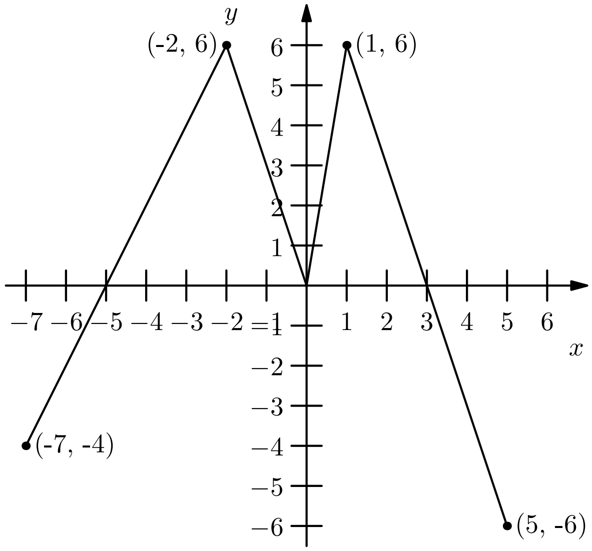
**19** The graph of the function $f$ is shown below. How many solutions does the equation $f(f(x)) = 6$ have?




**(A)** $2$ **(B)** $4$ **(C)** $5$ **(D)** $6$ **(E)** $7$

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**20** Suppose that $a$ and $b$ are digits, not both nine and not both zero, and the repeating decimal $0.\overline{ab}$ is expressed as a fraction in lowest terms. How many different denominators are possible?
**(A)** $3$ **(B)** $4$ **(C)** $5$ **(D)** $8$ **(E)** $9$

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**21** Consider the sequence of numbers: $4, 7, 1, 8, 9, 7, 6, \dots$. For $n > 2$, the $n$th term of the sequence is the units digit of the sum of the two previous terms. Let $S_n$ denote the sum of the first $n$ terms of this sequence. The smallest value of $n$ for which $S_n > 10,000$ is:
**(A)** $1992$ **(B)** $1999$ **(C)** $2001$ **(D)** $2002$ **(E)** $2004$

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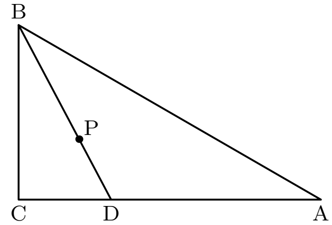
**22** Triangle $ABC$ is a right triangle with $\angle ACB$ as its right angle, $m\angle ABC = 60^\circ$, and $AB = 10$. Let $P$ be randomly chosen inside $\triangle ABC$, and extend $\overline{BP}$ to meet $\overline{AC}$ at $D$. What is the probability that $BD > 5\sqrt{2}$?



**(A)** $\frac{2-\sqrt{2}}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3-\sqrt{3}}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{5-\sqrt{5}}{5}$

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**23** In triangle $ABC$, side $AC$ and the perpendicular bisector of $BC$ meet in point $D$, and $BD$ bisects $\angle ABC$. If $AD = 9$ and $DC = 7$, what is the area of triangle $ABD$?
**(A)** $14$ **(B)** $21$ **(C)** $28$ **(D)** $14\sqrt{5}$ **(E)** $28\sqrt{5}$

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**24** Find the number of ordered pairs of real numbers $(a, b)$ such that $(a+bi)^{2002} = a - bi$.
**(A)** $1001$ **(B)** $1002$ **(C)** $2001$ **(D)** $2002$ **(E)** $2004$

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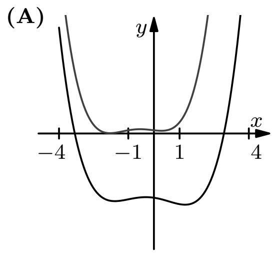
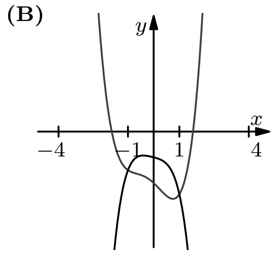
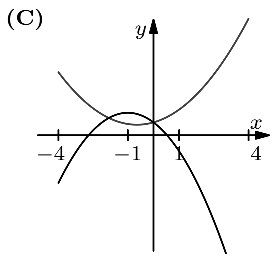
**25** The nonzero coefficients of a polynomial $P$ with real coefficients are all replaced by their mean to form a polynomial $Q$. Which of the following could be a graph of $y = P(x)$ and $y = Q(x)$ over the interval $-4 \le x \le 4$?



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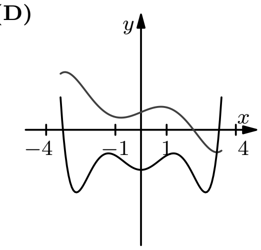
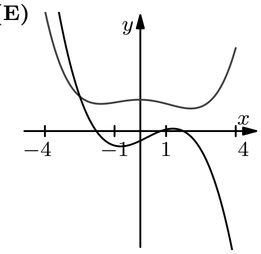



(E)

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|---|---|
| **–** | B |
| **1** | The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, ..., 999999999\}$ is a $9$-digit number $M$, all of whose digits are distinct. The number $M$ does not contain the digit<br>**(A)** $0$ **(B)** $2$ **(C)** $4$ **(D)** $6$ **(E)** $8$ |
| **2** | What is the value of $$(3x-2)(4x+1)-(3x-2)4x+1$$ when $x=4$?<br>**(A)** $0$ **(B)** $1$ **(C)** $10$ **(D)** $11$ **(E)** $12$ |
| **3** | For how many positive integers $n$ is $n^2-3n+2$ a prime number?<br>**(A)** none **(B)** one **(C)** two **(D)** more than two, but finitely many<br>**(E)** infinitely many |
| **4** | Let $n$ be a positive integer such that $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{n}$ is an integer. Which of the following statements is **not** true?<br>**(A)** $2$ divides $n$ **(B)** $3$ divides $n$ **(C)** $6$ divides $n$ **(D)** $7$ divides $n$<br>**(E)** $n>84$ |
| **5** | Let $v$, $w$, $x$, $y$, and $z$ be the degree measures of the five angles of a pentagon. Suppose $v<w<x<y<z$ and $v$, $w$, $x$, $y$, and $z$ form an arithmetic sequence. Find the value of $x$.<br>**(A)** $72$ **(B)** $84$ **(C)** $90$ **(D)** $108$ **(E)** $120$ |

**6** Suppose that $a$ and $b$ are are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions $a$ and $b$. Then the pair $(a, b)$ is

**(A)** $(-2, 1)$ **(B)** $(-1, 2)$ **(C)** $(1, -2)$ **(D)** $(2, -1)$ **(E)** $(4, 4)$

**7** The product of three consecutive positive integers is $8$ times their sum. What is the sum of their squares?
**(A)** $50$ **(B)** $77$ **(C)** $110$ **(D)** $149$ **(E)** $194$

**8** Suppose July of year $N$ has five Mondays. Which of the following must occur five times in August of year $N$? (Note: Both months have $31$ days.)
**(A)** Monday **(B)** Tuesday **(C)** Wednesday **(D)** Thursday **(E)** Friday

**9** If $a$, $b$, $c$, and $d$ are positive real numbers such that $a, b, c, d$ form an increasing arithmetic sequence and $a$, $b$, $d$ form a geometric sequence, then $\frac{a}{d}$ is
**(A)** $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

**10** How many different integers can be expressed as the sum of three distinct members of the set $\{1, 4, 7, 10, 13, 16, 19\}$?
**(A)** $13$ **(B)** $16$ **(C)** $24$ **(D)** $30$ **(E)** $35$

**11** The positive integers $A$, $B$, $A - B$, and $A + B$ are all prime numbers. The sum of these four primes is
**(A)** even **(B)** divisible by $3$ **(C)** divisible by $5$ **(D)** divisible by $7$
**(E)** prime

**12** For how many integers $n$ is $\frac{n}{20-n}$ the square of an integer?
**(A)** $1$ **(B)** $2$ **(C)** $3$ **(D)** $4$ **(E)** $10$

**13** The sum of $18$ consecutive positive integers is a perfect square. The smallest possible value of this sum is
**(A)** $169$ **(B)** $225$ **(C)** $289$ **(D)** $361$ **(E)** $441$

**14** Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?
**(A)** $8$ **(B)** $9$ **(C)** $10$ **(D)** $12$ **(E)** $16$

**15** How many four-digit numbers $N$ have the property that the three-digit number obtained by removing the leftmost digit is one ninth of $N$?
**(A)** $4$ **(B)** $5$ **(C)** $6$ **(D)** $7$ **(E)** $8$

 

**16** Juan rolls a fair regular octahedral die marked with the numbers $1$ through $8$. Then Amal rolls a fair six-sided die. What is the probability that the product of the two rolls is a multiple of $3$?
**(A)** $\frac{1}{12}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{2}{3}$

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**17** Andy's lawn has twice as much area as Beth's lawn and three times as much area as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower. If they all start to mow their lawns at the same time, who will finish first?

**(A)** Andy **(B)** Beth **(C)** Carlos **(D)** Andy and Carlos tie for first. **(E)** All three tie.

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**18** A point $P$ is randomly selected from the rectangular region with vertices $(0,0)$, $(2,0)$, $(2,1)$, $(0,1)$. What is the probability that $P$ is closer to the origin than it is to the point $(3,1)$?
**(A)** $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{5}$ **(E)** $1$

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**19** If $a, b$, and $c$ are positive real numbers such that $a(b+c)=152$, $b(c+a)=162$, and $c(a+b)=170$, then abc is
**(A)** $672$ **(B)** $688$ **(C)** $704$ **(D)** $720$ **(E)** $750$

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**20** Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90^\circ$. Let $M$ and $N$ be the midpoints of legs $OX$ and $OY$, respectively. Given that $XN = 19$ and $YM = 22$, find $XY$.
**(A)** $24$ **(B)** $26$ **(C)** $28$ **(D)** $30$ **(E)** $32$

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**21** For all positive integers $n$ less than $2002$, let

$$a_n = \begin{cases} 11 & \text{if } n \text{ is divisible by } 13 \text{ and } 14 \\ 13 & \text{if } n \text{ is divisible by } 11 \text{ and } 14 \\ 14 & \text{if } n \text{ is divisible by } 11 \text{ and } 13 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $\sum_{n=1}^{2001} a_n$.
**(A)** $448$ **(B)** $486$ **(C)** $1560$ **(D)** $2001$ **(E)** $2002$

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**22** For all integers $n$ greater than $1$, define $a_n = \frac{1}{\log_n 2002}$. Let $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then $b - c$ equals
**(A)** $-2$ **(B)** $-1$ **(C)** $\frac{1}{2002}$ **(D)** $\frac{1}{1001}$ **(E)** $\frac{1}{2}$

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**23** In $\triangle ABC$, we have $AB = 1$ and $AC = 2$. Side $BC$ and the median from $A$ to $BC$ have the same length. What is $BC$?
**(A)** $\frac{1+\sqrt{2}}{2}$ **(B)** $\frac{1+\sqrt{3}}{2}$ **(C)** $\sqrt{2}$ **(D)** $\frac{3}{2}$ **(E)** $\sqrt{3}$

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**24** A convex quadrilateral $ABCD$ with area $2002$ contains a point $P$ in its interior such that $PA = 24$, $PB = 32$, $PC = 28$, and $PD = 45$. FInd the perimeter of $ABCD$.
**(A)** $4\sqrt{2002}$ **(B)** $2\sqrt{8465}$ **(C)** $2\left(48+\sqrt{2002}\right)$ **(D)** $2\sqrt{8633}$ **(E)** $4\left(36+\sqrt{113}\right)$

**25** Let $f(x) = x^2 + 6x + 1$, and let $R$ denote the set of points $(x, y)$ in the coordinate plane such that

$$f(x) + f(y) \le 0 \text{ and } f(x) - f(y) \le 0$$

The area of $R$ is closest to
**(A)** $21$ **(B)** $22$ **(C)** $23$ **(D)** $24$ **(E)** $25$

**–** P

**-1** This test and the matching AMC 10P were developed for the use of a group of Taiwan schools, in early January of 2002. When Taiwan had taken the contests, the AMC released the questions here as a set of practice questions for the 2002 AMC 10 and AMC 12 contests.

**1** Which of the following numbers is a perfect square?

**(A)** $4^4 5^5 6^6$ **(B)** $4^4 5^6 6^5$ **(C)** $4^5 5^4 6^6$ **(D)** $4^6 5^4 6^5$ **(E)** $4^6 5^5 6^4$

**2** The function $f$ is given by the table

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 4 | 1 | 3 | 5 | 2 |

If $u_0 = 4$ and $u_{n+1} = f(u_n)$ for $n \ge 0$, find $u_{2002}$.

**(A)** $1$ **(B)** $2$ **(C)** $3$ **(D)** $4$ **(E)** $5$

**3** The dimensions of a rectangular box in inches are all positive integers and the volume of the box is $2002$ in$^3$. Find the minimum possible sum in inches of the three dimensions.

**(A)** $36$ **(B)** $38$ **(C)** $42$ **(D)** $44$ **(E)** $92$

**4** Let $a$ and $b$ be distinct real numbers for which

$$\frac{a}{b} + \frac{a + 10b}{b + 10a} = 2.$$

Find $\frac{a}{b}$.

**(A)** $0.6$ **(B)** $0.7$ **(C)** $0.8$ **(D)** $0.9$ **(E)** $1$

**5** For how many positive integers $m$ is

$$\frac{2002}{m^2 - 2}$$

a positive integer?

**(A)** one **(B)** two **(C)** three **(D)** four **(E)** more than four

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**6** Participation in the local soccer league this year is $10\%$ higher than last year. The number of males increased by $5\%$ and the number of females increased by $20\%$. What fraction of the soccer league is now female?

**(A)** $\frac{1}{3}$ **(B)** $\frac{4}{11}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{4}{9}$ **(E)** $\frac{1}{2}$

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**7** How many three-digit numbers have at least one $2$ and at least one $3$?

**(A)** $52$ **(B)** $54$ **(C)** $56$ **(D)** $58$ **(E)** $60$

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**8** Let $AB$ be a segment of length $26$, and let points $C$ and $D$ be located on $AB$ such that $AC = 1$ and $AD = 8$. Let $E$ and $F$ be points on one of the semicircles with diameter $AB$ for which $EC$ and $FD$ are perpendicular to $AB$. Find $EF$.

**(A)** $5$ **(B)** $5\sqrt{2}$ **(C)** $7$ **(D)** $7\sqrt{2}$ **(E)** $12$

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**9** Two walls and the ceiling of a room meet at right angles at point $P$. A fly is in the air one meter from one wall, eight meters from the other wall, and $9$ meters from point $P$. How many meters is the fly from the ceiling?

**(A)** $\sqrt{13}$ **(B)** $\sqrt{14}$ **(C)** $\sqrt{15}$ **(D)** $4$ **(E)** $\sqrt{17}$

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**10** Let $f_n(x) = \sin^n x + \cos^n x$. For how many $x$ in $[0, \pi]$ is it true that

$$6f_4(x) - 4f_6(x) = 2f_2(x)?$$

**(A)** $2$ **(B)** $4$ **(C)** $6$ **(D)** $8$ **(E)** more than 8

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**11** Let $t_n = \frac{n(n+1)}{2}$ be the $n$th triangular number. Find

$$\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \cdots + \frac{1}{t_{2002}}.$$

**(A)** $\frac{4003}{2003}$ **(B)** $\frac{2001}{1001}$ **(C)** $\frac{4004}{2003}$ **(D)** $\frac{4001}{2001}$ **(E)** $2$

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**12** For how many positive integers $n$ is $n^3 - 8n^2 + 20n - 13$ a prime number?

**(A)** $1$ **(B)** $2$ **(C)** $3$ **(D)** $4$ **(E)** more than 4

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**13** What is the maximum value of $n$ for which there is a set of distinct positive integers $k_1, k_2, \ldots, k_n$ for which

$$k_1^2 + k_2^2 + \ldots + k_n^2 = 2002?$$

**(A)** $14$ **(B)** $15$ **(C)** $16$ **(D)** $17$ **(E)** $18$

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**14** Find $i + 2i^2 + 3i^3 + \ldots + 2002i^{2002}$.

**(A)** $-999 + 1002i$ **(B)** $-1002 + 999i\sqrt{2}$ **(C)** $-1001 + 1000i$ **(D)** $-1002 + 1001i$ **(E)** $i$

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**15** There are $1001$ red marbles and $1001$ black marbles in a box. Let $P_s$ be the probability that two marbles drawn at random from the box are the same color, and let $P_d$ be the probability that they are different colors. Find $|P_s - P_d|$.

**(A)** $0$ **(B)** $\frac{1}{2002}$ **(C)** $\frac{1}{2001}$ **(D)** $\frac{2}{2001}$ **(E)** $\frac{1}{1000}$

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**16** The altitudes of a triangles are $12$, $15$, and $20$. The largest angle in this triangle is

**(A)** $72^\circ$ **(B)** $75^\circ$ **(C)** $90^\circ$ **(D)** $108^\circ$ **(E)** $120^\circ$

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**17** Let $f(x) = \sqrt{\sin^4 x + 4\cos^2 x} - \sqrt{\cos^4 x + 4\sin^2 x}$. An equivalent form of $f(x)$ is

**(A)** $1 - \sqrt{2}\sin x$ **(B)** $-1 + \sqrt{2}\cos x$ **(C)** $\cos\frac{x}{2} - \sin\frac{x}{2}$ **(D)** $\cos x - \sin x$ **(E)** $\cos 2x$

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**18** If $a, b, c$ are real numbers such that $a^2 + 2b = 7$, $b^2 + 4c = -7$, and $c^2 + 6a = -14$, find $a^2 + b^2 + c^2$.

**(A)** $14$ **(B)** $21$ **(C)** $28$ **(D)** $35$ **(E)** $49$

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**19** In quadrilateral $ABCD$, $m\angle B = m\angle C = 120^\circ$, $AB = 3$, $BC = 4$, and $CD = 5$. Find the area of $ABCD$.

**(A)** $15$ **(B)** $9\sqrt{3}$ **(C)** $\frac{45\sqrt{3}}{4}$ **(D)** $\frac{47\sqrt{3}}{4}$ **(E)** $15\sqrt{3}$

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**20** Let $f$ be a real-valued function such that

$$f(x) + 2f\left(\frac{2002}{x}\right) = 3x$$

for all $x > 0$. Find $f(2)$.

**(A)** $1000$ **(B)** $2000$ **(C)** $3000$ **(D)** $4000$ **(E)** $6000$

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**21** Let $a$ and $b$ be real numbers greater than $1$ for which there exists a positive real number $c$, different from $1$, such that

$$2(\log_a c + \log_b c) = 9\log_{ab} c.$$

Find the largest possible value of $\log_a b$.

**(A)** $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $2$ **(D)** $\sqrt{6}$ **(E)** $3$

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**22** Under the new AMC 10, 12 scoring method, $6$ points are given for each correct answer, $2.5$ points are given for each unanswered question, and no points are given for an incorrect answer. Some of the possible scores between $0$ and $150$ can be obtained in only one way, for example, the only way to obtain a score of $146.5$ is to have 24 correct answers and one unanswered question. Some scores can be obtained in exactly two ways; for example, a score of $104.5$ can be obtained with $17$ correct answers, $1$ unanswered question, and $7$ incorrect, and also with $12$ correct answers and $13$ unanswered questions. There are three scores that can be obtained in exactly three ways. What is their sum?

**(A)** $175$ **(B)** $179.5$ **(C)** $182$ **(D)** $188.5$ **(E)** $201$

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**23** The equation $z(z+i)(z+3i)=2002i$ has a zero of the form $a+bi$, where $a$ and $b$ are positive real numbers. Find $a$.

**(A)** $\sqrt{118}$ **(B)** $\sqrt{210}$ **(C)** $2\sqrt{210}$ **(D)** $\sqrt{2002}$ **(E)** $100\sqrt{2}$

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**24** Let $ABCD$ be a regular tetrahedron and let $E$ be a point inside the face $ABC$. Denote by $s$ the sum of the distances from $E$ to the faces $DAB$, $DBC$, $DCA$, and by $S$ the sum of the distances from $E$ to the edges $AB$, $BC$, $CA$. Then $\frac{s}{S}$ equals

**(A)** $\sqrt{2}$ **(B)** $\frac{2\sqrt{2}}{3}$ **(C)** $\frac{\sqrt{6}}{2}$ **(D)** $2$ **(E)** $3$

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**25** Let $a$ and $b$ be real numbers such that $\sin a + \sin b = \frac{\sqrt{2}}{2}$ and $\cos a + \cos b = \frac{\sqrt{6}}{2}$. Find $\sin(a+b)$.

**(A)** $\frac{1}{2}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** $\frac{\sqrt{6}}{2}$ **(E)** $1$

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