problems 1, 2, 3, 4, 6

www.artofproblemsolving.com/community/c486732

by gausskarl, phuong, linqaszayi, tenplusten, parmenides51

– 1st Day

Problem 1 Let α be the positive root of the equation $x^2 + x = 5$. Let n be a positive integer number, and let $c_0, c_1, \ldots, c_n \in \mathbb{N}$ be such that $c_0 + c_1\alpha + c_2\alpha^2 + \cdots + c_n\alpha^n = 2015.$
a. Prove that $c_0 + c_1 + c_2 + \cdots + c_n \equiv 2 \pmod 3$.
b. Find the minimum value of the sum $c_0 + c_1 + c_2 + \cdots + c_n$.

Problem 2 sorry if this has been posted before .

given a fixed circle (O) and two fixed point B, C on it.point A varies on circle (O). let I be the midpoint of BC and H be the orthocenter of $\triangle ABC$. ray IH meet (O) at K ,AH meet BC at D ,KD meet (O) at M .a line pass M and perpendicular to BC meet AI at N.
a) prove that N varies on a fixed circle.
b) a circle pass N and tangent to AK at A cut AB, AC at P, Q. let J be the midpoint of PQ .prove that AJ pass through a fixed point.

Problem 3 A positive interger number k is called $t-m$-property if forall positive interger number a, there exists a positive integer number n such that $1^k + 2^k + 3^k + ... + n^k \equiv a (\mathrm{mod} m)$.
a) Find all positive integer numbers k which has $t-20$-property.
b) Find smallest positive integer number k which has $t-20^{15}$-property.

– 2nd Day

Problem 4 There are 100 students who praticipate at exam.Also there are 25 members of jury.Each student is checked by one jury.Known that every student likes 10 jury $a)$ Prove that we can select 7 jury such that any student likes at least one jury. $b)$ Prove that we can make this every student will be checked by the jury that he likes and every jury will check at most 10 students.

Problem 5 Let ABC be a triangle with an interior point P such that $\angle APB = \angle APC = \alpha$ and $\alpha > 180^o - \angle BAC$. The circumcircle of triangle APB cuts AC at E, the circumcircle of triangle APC cuts AB at F. Let Q be the point in the triangle AEF such that $\angle AQE = \angle AQF = \alpha$. Let D be the symmetric point of Q wrt EF. Angle bisector of $\angle EDF$ cuts AP at T.
a) Prove that $\angle DET = \angle ABC, \angle DFT = \angle ACB$.
b) Straight line PA cuts straight lines DE, DF at M, N respectively. Denote I, J the incenters of the triangles PEM, PFN, and K the circumcenter of the triangle DIJ. Straight line DT cut (K) at H. Prove that HK passes through the incenter of the triangle DMN.

Problem 6 Find the smallest positive interger number n such that there exists n real numbers $a_1, a_2, \dots, a_n$ satisfied three conditions as follow:
a. $a_1 + a_2 + \dots + a_n > 0$;
b. $a_1^3 + a_2^3 + \dots + a_n^3 < 0$;
c. $a_1^5 + a_2^5 + \dots + a_n^5 > 0$.