AMC 12/AHSME 2004

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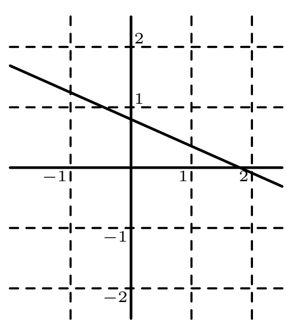
–	A
1	Alicia earns $\$20$ per hour, of which 1.45% is deducted to pay local taxes. How many cents per hour of Alicia's wages are used to pay local taxes? **(A)** 0.0029 **(B)** 0.029 **(C)** 0.29 **(D)** 2.9 **(E)** 29
2	On the AMC 12, each correct answer is worth 6 points, each incorrect answer is worth 0 points, and each problem left unanswered is worth 2.5 points. If Charlyn leaves 8 of the 25 problems unanswered, how many of the remaining problems must she answer correctly in order to score at least 100? **(A)** 11 **(B)** 13 **(C)** 14 **(D)** 16 **(E)** 17
3	For how many ordered pairs of positive integers (x, y) is $x + 2y = 100$? **(A)** 33 **(B)** 49 **(C)** 50 **(D)** 99 **(E)** 100
4	Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and grand-daughters have no children? **(A)** 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26
5	The graph of the line $y = mx + b$ is shown. Which of the following is true?  **(A)** $mb < -1$ **(B)** $-1 < mb < 0$ **(C)** $mb = 0$ **(D)** $0 < mb < 1$ **(E)** $mb > 1$
6	Let $U = 2 \times 2004^{2005}$, $V = 2004^{2005}$, $W = 2003 \times 2004^{2004}$, $X = 2 \times 2004^{2004}$, $Y = 2004^{2004}$ and $Z = 2004^{2003}$. Which of the following is the largest?

(A) $U-V$ **(B)** $V-W$ **(C)** $W-X$ **(D)** $X-Y$ **(E)** $Y-Z$

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7 A game is played with tokens according to the following rules. In each round, the player with the most tokens gives one token to each of the other players and also places one token into a discard pile. The game ends when some player runs out of tokens. Players A, B, and C start with 15, 14, and 13 tokens, respectively. How many rounds will there be in the game?
(A) 36 **(B)** 37 **(C)** 38 **(D)** 39 **(E)** 40

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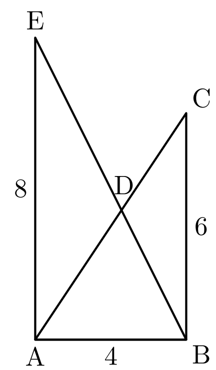
8 In the overlapping triangles $\triangle ABC$ and $\triangle ABE$ sharing common side AB, $\angle EAB$ and $\angle ABC$ are right angles, $AB=4$, $BC=6$, $AE=8$, and $\overline{AC}$ and $\overline{BE}$ intersect at D. What is the difference between the areas of $\triangle ADE$ and $\triangle BDC$?



(A) 2 **(B)** 4 **(C)** 5 **(D)** 8 **(E)** 9

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9 A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars would increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?
(A) 10% **(B)** 25% **(C)** 36% **(D)** 50% **(E)** 60%

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10 The sum of 49 consecutive integers is 7^5. What is their median?
(A) 7 **(B)** 7^2 **(C)** 7^3 **(D)** 7^4 **(E)** 7^5

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11 The average value of all the pennies, nickels, dimes, and quarters in Paula's purse is 20 cents. If she had one more quarter, the average value would be 21 cents. How many dimes does she have in her purse?
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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12 Let $A=(0,9)$ and $B=(0,12)$. Points A' and B' are on the line $y=x$, and $\overline{AA'}$ and $\overline{BB'}$ intersect at $C=(2,8)$. What is the length of $\overline{A'B'}$?
(A) 2 **(B)** $2\sqrt{2}$ **(C)** 3 **(D)** $2+\sqrt{2}$ **(E)** $3\sqrt{2}$

13 Let S be the set of points (a, b) in the coordinate plane, where each of a and b may be -1, 0, or 1. How many distinct lines pass through at least two members of S?
(A) 8 **(B)** 20 **(C)** 24 **(D)** 27 **(E)** 36

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14 A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term in the geometric progression?
(A) 1 **(B)** 4 **(C)** 36 **(D)** 49 **(E)** 81

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15 Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?
(A) 250 **(B)** 300 **(C)** 350 **(D)** 400 **(E)** 500

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16 The set of all real numbers x for which

$$\log_{2004}(\log_{2003}(\log_{2002}(\log_{2001} x)))$$

is defined is $\{x|x > c\}$. What is the value of c?

(A) 0 **(B)** 2001^{2002} **(C)** 2002^{2003} **(D)** 2003^{2004} **(E)** $2001^{2002^{2003}}$

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17 Let f be a function with the following properties:
(i) $f(1) = 1$, and
(ii) $f(2n) = n \times f(n)$, for any positive integer n.
What is the value of $f(2^{100})$?

(A) 1 **(B)** 2^{99} **(C)** 2^{100} **(D)** 2^{4950} **(E)** 2^{9999}

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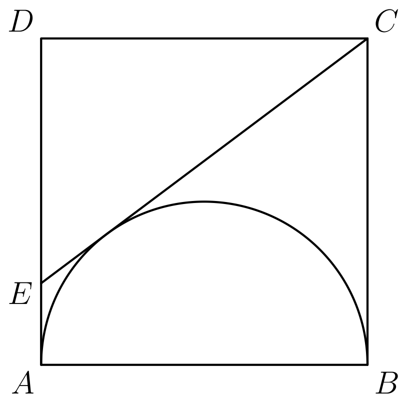
18 Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicricle from C intersects side $\overline{AD}$ at E. What is the length of $\overline{CE}$?



(A) $\frac{2+\sqrt{5}}{2}$ **(B)** $\sqrt{5}$ **(C)** $\sqrt{6}$ **(D)** $\frac{5}{2}$ **(E)** $5-\sqrt{5}$

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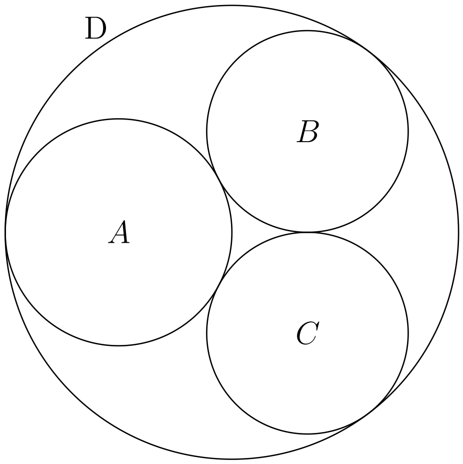
19 Circles A, B and C are externally tangent to each other and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D. What is the radius of circle B?



(A) $\frac{2}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{8}{9}$ **(E)** $\frac{1+\sqrt{3}}{3}$

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20 Select numbers a and b between 0 and 1 independently and at random, and let c be their sum. Let A, B and C be the results when a, b and c, respectively, are rounded to the nearest integer. What is the probability that $A+B=C$?
(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

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21 If $\sum_{n=0}^{\infty} \cos^{2n}\theta = 5$, what is the value of $\cos 2\theta$?

(A) $\frac{1}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{\sqrt{5}}{5}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{4}{5}$

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22 Three mutually tangent spheres of radius 1 rest on a horizontal plane. A sphere of radius 2 rests on them. What is the distance from the plane to the top of the larger sphere?

(A) $3 + \frac{\sqrt{30}}{2}$ **(B)** $3 + \frac{\sqrt{69}}{3}$ **(C)** $3 + \frac{\sqrt{123}}{4}$ **(D)** $\frac{52}{9}$ **(E)** $3 + 2\sqrt{2}$

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23 A polynomial

$$P(x) = c_{2004}x^{2004} + c_{2003}x^{2003} + ... + c_1x + c_0$$

has real coefficients with $c_{2004} \neq 0$ and 2004 distinct complex zeroes $z_k = a_k + b_ki$, $1 \leq k \leq 2004$ with a_k and b_k real, $a_1 = b_1 = 0$, and

$$\sum_{k=1}^{2004} a_k = \sum_{k=1}^{2004} b_k.$$

Which of the following quantities can be a nonzero number?

(A) c_0 **(B)** c_{2003} **(C)** $b_2b_3...b_{2004}$ **(D)** $\sum_{k=1}^{2004} a_k$ **(E)** $\sum_{k=1}^{2004} c_k$

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24 A plane contains points A and B with $AB = 1$. Let S be the union of all disks of radius 1 in the plane that cover $\overline{AB}$. What is the area of S?

(A) $2\pi + \sqrt{3}$ **(B)** $\frac{8\pi}{3}$ **(C)** $3\pi - \frac{\sqrt{3}}{2}$ **(D)** $\frac{10\pi}{3} - \sqrt{3}$ **(E)** $4\pi - 2\sqrt{3}$

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25 For each integer $n \geq 4$, let a_n denote the base-n number $0.\overline{133}_n$. The product $a_4a_5 \cdots a_{99}$ can be expressed as $\frac{m}{n!}$, where m and n are positive integers and n is as small as possible. What is the value of m?

(A) 98 **(B)** 101 **(C)** 132 **(D)** 798 **(E)** 962

– B

– February 25th

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1 At each basketball practice last week, Jenny made twice as many free throws as she made at the previous practice. At her fifth practice she made 48 free throws. How many free throws did she make at the first practice?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 15

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2 In the expression $c \cdot a^b - d$, the values of a, b, c, and d are 0, 1, 2, and 3, although not necessarily in that order. What is the maximum possible value of the result?

(A) 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 10

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3 If x and y are positive integers for which $2^x3^y = 1296$, what is the value of $x + y$?
(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

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4 An integer x, with $10 \leq x \leq 99$, is to be chosen. If all choices are equally likely, what is the probability that at least one digit of x is a 7?
(A) $\frac{1}{9}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{19}{90}$ **(D)** $\frac{2}{9}$ **(E)** $\frac{1}{3}$

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5 On a trip from the United States to Canada, Isabella took d U.S. dollars. At the border she exchanged them all, receiving 10 Canadian dollars for every 7 U.S. dollars. After spending 60 Canadian dollars, she had d Canadian dollars left. What is the sum of the digits of d?
(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

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6 Minneapolis-St. Paul International Airport is 8 miles southwest of downtown St. Paul and 10 miles southeast of downtown Minneapolis. Which of the following is closest to the number of miles between downtown St. Paul and downtown Minneapolis?
(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

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7 A square has sides of length 10, and a circle centered at one of its vertices has radius 10. What is the area of the union of the regions enclosed by the square and the circle?
(A) $200 + 25\pi$ **(B)** $100 + 75\pi$ **(C)** $75 + 100\pi$ **(D)** $100 + 100\pi$ **(E)** $100 + 125\pi$

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8 A grocer makes a display of cans in which the top row has one can and each lower row has two more cans than the row above it. If the display contains 100 cans, how many rows does it contain?
(A) 5 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

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9 The point $(-3, 2)$ is rotated 90° clockwise around the origin to point B. Point B is then reflected over the line $y = x$ to point C. What are the coordinates of C?
(A) $(-3, -2)$ **(B)** $(-2, -3)$ **(C)** $(2, -3)$ **(D)** $(2, 3)$ **(E)** $(3, 2)$

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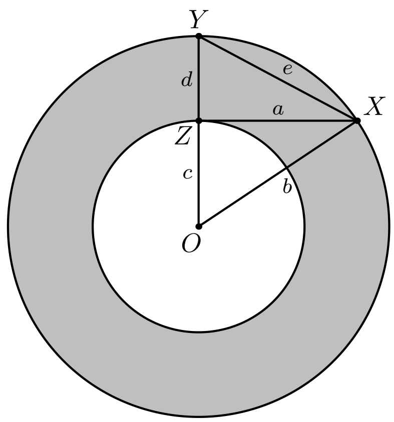
10 An *annulus* is the region between two concentric circles. The concentric circles in the figure have radii b and c, with $b > c$. Let $\overline{OX}$ be a radius of the larger circle, let $\overline{XZ}$ be tangent to the smaller circle at Z, and let $\overline{OY}$ be the radius of the larger circle that contains Z. Let $a = XZ$, $d = YZ$, and $e = XY$. What is the area of the annulus?
(A) πa^2 **(B)** πb^2 **(C)** πc^2 **(D)** πd^2 **(E)** πe^2



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11 All the students in an algebra class took a 100-point test. Five students scored 100, each student scored at least 60, and the mean score was 76. What is the smallest possible number of students in the class?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

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12 In the sequence $2001, 2002, 2003, \ldots$, each term after the third is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is $2001 + 2002 - 2003 = 2000$. What is the 2004^{th} term in this sequence?

(A) -2004 **(B)** -2 **(C)** 0 **(D)** 4003 **(E)** 6007

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13 If $f(x) = ax + b$ and $f^{-1}(x) = bx + a$ with a and b real, what is the value of $a + b$?

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

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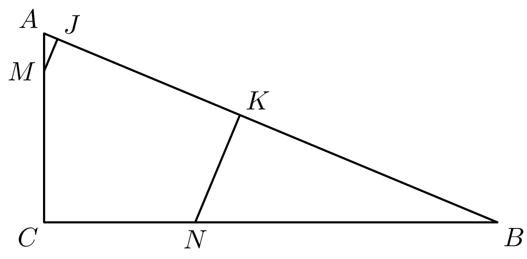
14 In $\triangle ABC$, $AB = 13$, $AC = 5$, and $BC = 12$. Points M and N lie on $\overline{AC}$ and $\overline{BC}$, respectively, with $CM = CN = 4$. Points J and K are on $\overline{AB}$ so that $\overline{MJ}$ and $\overline{NK}$ are perpendicular to $\overline{AB}$. What is the area of pentagon $CMJKN$?



(A) 15 **(B)** $\frac{81}{5}$ **(C)** $\frac{205}{12}$ **(D)** $\frac{240}{13}$ **(E)** 20

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15 The two digits in Jack's age are the same as the digits in Bill's age, but in reverse order. In five years Jack will be twice as old as Bill will be then. What is the difference in their current ages?

(A) 9 **(B)** 18 **(C)** 27 **(D)** 36 **(E)** 45

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16 A function f is defined by $f(z) = i\bar{z}$, where $i = \sqrt{-1}$ and $\bar{z}$ is the complex conjugate of z. How many values of z satisfy both $|z| = 5$ and $f(z) = z$?
(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

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17 For some real numbers a and b, the equation

$$8x^3 + 4ax^2 + 2bx + a = 0$$

has three distinct positive roots. If the sum of the base-2 logarithms of the roots is 5, what is the value of a?
(A) -256 **(B)** -64 **(C)** -8 **(D)** 64 **(E)** 256

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18 Points A and B are on the parabola $y = 4x^2 + 7x - 1$, and the origin is the midpoint of $\overline{AB}$. What is the length of $\overline{AB}$?
(A) $2\sqrt{5}$ **(B)** $5 + \frac{\sqrt{2}}{2}$ **(C)** $5 + \sqrt{2}$ **(D)** 7 **(E)** $5\sqrt{2}$

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19 A truncated cone has horizontal bases with radii 18 and 2. A sphere is tangent to the top, bottom, and lateral surface of the truncated cone. What is the radius of the sphere?
(A) 6 **(B)** $4\sqrt{5}$ **(C)** 9 **(D)** 10 **(E)** $6\sqrt{3}$

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20 Each face of a cube is painted either red or blue, each with probability $1/2$. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?
(A) $\frac{1}{4}$ **(B)** $\frac{5}{16}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

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21 The graph of $2x^2 + xy + 3y^2 - 11x - 20y + 40 = 0$ is an ellipse in the first quadrant of the xy-plane. Let a and b be the maximum and minimum values of $\frac{y}{x}$ over all points (x, y) on the ellipse. What is the value of $a + b$?
(A) 3 **(B)** $\sqrt{10}$ **(C)** $\frac{7}{2}$ **(D)** $\frac{9}{2}$ **(E)** $2\sqrt{14}$

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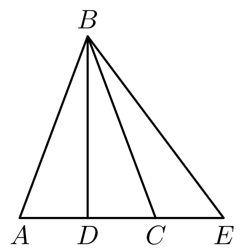
22 The square

50	*b*	*c*
d	*e*	*f*
g	*h*	2

is a multiplicative magic square. That is, the product of the numbers in each row, column, and diagonal is the same. If all the entries are positive integers, what is the sum of the possible values of g?
(A) 10 **(B)** 25 **(C)** 35 **(D)** 62 **(E)** 136

23 The polynomial $x^3 - 2004x^2 + mx + n$ has integer coefficients and three distinct positive zeros. Exactly one of these is an integer, and it is the sum of the other two. How many values of n are possible?
(A) $250{,}000$ **(B)** $250{,}250$ **(C)** $250{,}500$ **(D)** $250{,}750$ **(E)** $251{,}000$

24 In $\triangle ABC$, $AB = BC$, and BD is an altitude. Point E is on the extension of $\overline{AC}$ such that $BE = 10$. The values of $\tan CBE$, $\tan DBE$, and $\tan ABE$ form a geometric progression, and the values of $\cot DBE$, $\cot CBE$, $\cot DBC$ form an arithmetic progression. What is the area of $\triangle ABC$?



(A) 16 **(B)** $\frac{50}{3}$ **(C)** $10\sqrt{3}$ **(D)** $8\sqrt{5}$ **(E)** 18

25 Given that 2^{2004} is a 604-digit number whose first digit is 1, how many elements of the set $S = \{2^0, 2^1, 2^2, \ldots, 2^{2003}\}$ have a first digit of 4?
(A) 194 **(B)** 195 **(C)** 196 **(D)** 197 **(E)** 198

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