AMC 12/AHSME 2012

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– A

1 A bug crawls along a number line, starting at -2. It crawls to -6, then turns around and crawls to 5. How many units does the bug crawl altogether?

(A) 9 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 15

2 Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?

(A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

3 A box 2 centimeters high, 3 centimeters wide, and 5 centimeters long can hold 40 grams of clay. A second box with twice the height, three times the width, and the same length as the first box can hold n grams of clay. What is n?

(A) 120 **(B)** 160 **(C)** 200 **(D)** 240 **(E)** 280

4 In a bag of marbles, $\frac{3}{5}$ of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

(A) $\frac{2}{5}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{4}{7}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{4}{5}$

5 A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries. How many cherries are there in the fruit salad?

(A) 8 **(B)** 16 **(C)** 25 **(D)** 64 **(E)** 96

6 The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

7 Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle?

(A) 5 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

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8 An *iterative average* of the numbers 1, 2, 3, 4, and 5 is computed in the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure?

(A) $\frac{31}{16}$ **(B)** 2 **(C)** $\frac{17}{8}$ **(D)** 3 **(E)** $\frac{65}{16}$

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9 A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not by 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?

(A) Friday **(B)** Saturday **(C)** Sunday **(D)** Monday **(E)** Tuesday

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10 A triangle has area 30, one side of length 10, and the median to that side of length 9. Let θ be the acute angle formed by that side and the median. What is $\sin\theta$?

(A) $\frac{3}{10}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{9}{20}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{9}{10}$

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11 Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is $\frac{1}{2}$, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round?

(A) $\frac{5}{72}$ **(B)** $\frac{5}{36}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{3}$ **(E)** 1

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12 A square region $ABCD$ is externally tangent to the circle with equation $x^2+y^2=1$ at the point $(0,1)$ on the side CD. Vertices A and B are on the circle with equation $x^2+y^2=4$. What is the side length of this square?

(A) $\frac{\sqrt{10}+5}{10}$ **(B)** $\frac{2\sqrt{5}}{5}$ **(C)** $\frac{2\sqrt{2}}{3}$ **(D)** $\frac{2\sqrt{19}-4}{5}$ **(E)** $\frac{9-\sqrt{17}}{5}$

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13 Paula the painter and her two helpers each paint at constant, but different, rates. They always start at 8:00 AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at 4:00 PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at 2:12 PM. On Wednesday Paula worked by herself and finished the house by working until 7:12 PM. How long, in minutes, was each day's lunch break?

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 60

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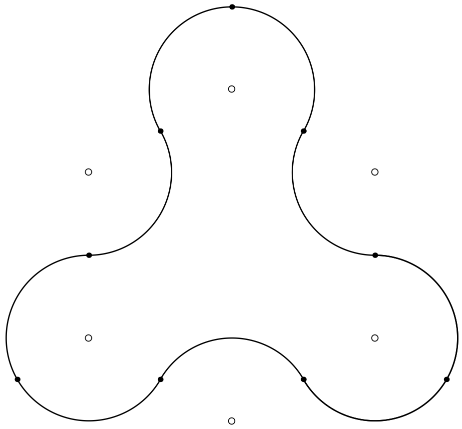
14 The closed curve in the figure is made up of 9 congruent circular arcs each of length $\frac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?



(A) $2\pi + 6$ **(B)** $2\pi + 4\sqrt{3}$ **(C)** $3\pi + 4$ **(D)** $2\pi + 3\sqrt{3} + 2$ **(E)** $\pi + 6\sqrt{3}$

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15 A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random. The square is the rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability that the grid is now entirely black?

(A) $\frac{49}{512}$ **(B)** $\frac{7}{64}$ **(C)** $\frac{121}{1024}$ **(D)** $\frac{81}{512}$ **(E)** $\frac{9}{32}$

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16 Circle C_1 has its center O lying on circle C_2. The two circles meet at X and Y. Point Z in the exterior of C_1 lies on circle C_2 and $XZ = 13$, $OZ = 11$, and $YZ = 7$. What is the radius of circle C_1?

(A) 5 **(B)** $\sqrt{26}$ **(C)** $3\sqrt{3}$ **(D)** $2\sqrt{7}$ **(E)** $\sqrt{30}$

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17 Let S be a subset of $\{1, 2, 3, \ldots, 30\}$ with the property that no pair of distinct elements in S has a sum divisible by 5. What is the largest possible size of S?

(A) 10 **(B)** 13 **(C)** 15 **(D)** 16 **(E)** 18

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18 Triangle ABC has $AB = 27$, $AC = 26$, and $BC = 25$. Let I denote the intersection of the internal angle bisectors of $\triangle ABC$. What is BI?

(A) 15 **(B)** $5 + \sqrt{26} + 3\sqrt{3}$ **(C)** $3\sqrt{26}$ **(D)** $\frac{2}{3}\sqrt{546}$ **(E)** $9\sqrt{3}$

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19 Adam, Benin, Chiang, Deshawn, Esther, and Fiona have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen?

(A) 60 **(B)** 170 **(C)** 290 **(D)** 320 **(E)** 660

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20 Consider the polynomial

$$P(x) = \prod_{k=0}^{10}(x^{2^k} + 2^k) = (x+1)(x^2+2)(x^4+4)\cdots(x^{1024}+1024).$$

The coefficient of x^{2012} is equal to 2^a. What is a?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 10 **(E)** 24

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21 Let $a, b,$ and c be positive integers with $a \geq b \geq c$ such that

$$a^2 - b^2 - c^2 + ab = 2011 \text{ and}$$
$$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997$$

What is a?

(A) 249 **(B)** 250 **(C)** 251 **(D)** 252 **(E)** 253

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22 Distinct planes $p_1, p_2, ..., p_k$ intersect the interior of a cube Q. Let S be the union of the faces of Q and let $P = \bigcup_{j=1}^{k} p_j$. The intersection of P and S consists of the union of all segments joining the midpoints of every pair of edges belonging to the same face of Q. What is the difference between the maximum and minimum possible values of k?

(A) 8 **(B)** 12 **(C)** 20 **(D)** 23 **(E)** 24

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23 Let S be the square one of whose diagonals has endpoints $(0.1, 0.7)$ and $(-0.1, -0.7)$. A point $v = (x, y)$ is chosen uniformly at random over all pairs of real numbers x and y such that $0 \leq x \leq 2012$ and $0 \leq y \leq 2012$. Let $T(v)$ be a translated copy of S centered at v. What is the probability that the square region determined by $T(v)$ contains exactly two points with integer coordinates in its interior?

(A) 0.125 **(B)** 0.14 **(C)** 0.16 **(D)** 0.25 **(E)** 0.32

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24 Let $\{a_k\}_{k=1}^{2011}$ be the sequence of real numbers defined by

$$a_1 = 0.201, \quad a_2 = (0.2011)^{a_1}, \quad a_3 = (0.20101)^{a_2}, \quad a_4 = (0.201011)^{a_3},$$

and more generally

$$a_k = \begin{cases} (0.\underbrace{20101\cdots 0101}_{k+2 \text{ digits}})^{a_{k-1}}, & \text{if } k \text{ is odd,} \\ (0.\underbrace{20101\cdots 01011}_{k+2 \text{ digits}})^{a_{k-1}}, & \text{if } k \text{ is even.} \end{cases}$$

Rearranging the numbers in the sequence $\{a_k\}_{k=1}^{2011}$ in decreasing order produces a new sequence $\{b_k\}_{k=1}^{2011}$. What is the sum of all the integers k, $1 \le k \le 2011$, such that $a_k = b_k$?

(A) 671 **(B)** 1006 **(C)** 1341 **(D)** 2011 **(E)** 2012

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25 Let $f(x) = |2\{x\} - 1|$ where $\{x\}$ denotes the fractional part of x. The number n is the smallest positive integer such that the equation

$$nf(xf(x)) = x$$

has at least 2012 real solutions x. What is n?

Note: the fractional part of x is a real number $y = \{x\}$, such that $0 \le y < 1$ and $x - y$ is an integer.

(A) 30 **(B)** 31 **(C)** 32 **(D)** 62 **(E)** 64

– B

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1 Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

(A) 48 **(B)** 56 **(C)** 64 **(D)** 72 **(E)** 80

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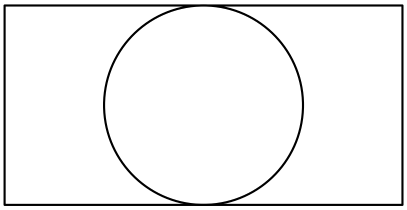
2 A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the the length of the rectangle to its width is $2 : 1$. What is the area of the rectangle?



(A) 50 **(B)** 100 **(C)** 125 **(D)** 150 **(E)** 200

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3 For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same
number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 54

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4 Suppose that the euro is worth 1.30 dollars. If Diana has 500 dollars and Etienne has 400 euros, by what percent is the value of Etienne's money greater than the value of Diana's money?

(A) 2 **(B)** 4 **(C)** 6.5 **(D)** 8 **(E)** 13

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5 Two integers have a sum of 26. When two more integers are added to the first two integers the sum is 41. Finally when two more integers are added to the sum of the previous four integers the sum is 57. What is the minimum number of even integers among the 6 integers?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

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6 In order to estimate the value of $x - y$ where x and y are real numbers with $x > y > 0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her values. Which of the following statements is necessarily correct?

(A) Her estimate is larger than $x - y$
(B) Her estimate is smaller than $x - y$
(C) Her estimate equals $x - y$
(D) Her estimate equals $y - x$
(E) Her estimate is 0

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7 Small lights are hung on a string 6 inches apart in the order red, red, green, green, green, red, red, green, green, green, and so on continuing this pattern of 2 red lights followed by 3 green lights. How many feet separate the third red light and the 21st red light?

Note: 1 foot is equal to 12 inches.

(A) 18 **(B)** 18.5 **(C)** 20 **(D)** 20.5 **(E)** 22.5

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8 A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

(A) 729 **(B)** 972 **(C)** 1024 **(D)** 2187 **(E)** 2304

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9 It takes Clea 60 seconds to walk down an escalator when it is not operating and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

(A) 36 **(B)** 40 **(C)** 42 **(D)** 48 **(E)** 52

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10 What is the area of the polygon whose vertices are the points of intersection of the curves $x^2 + y^2 = 25$ and $(x - 4)^2 + 9y^2 = 81$?

(A) 24 **(B)** 27 **(C)** 36 **(D)** 37.5 **(E)** 42

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11 In the equation below, A and B are consecutive positive integers, and A, B, and $A+B$ represent number bases:

$$132_A + 43_B = 69_{A+B}.$$

What is $A+B$?

(A) 9 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 17

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12 How many sequences of zeros and ones of length 20 have all the zeros consecutive, or all the ones consecutive, or both?

(A) 190 **(B)** 192 **(C)** 211 **(D)** 380 **(E)** 382

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13 Two parabolas have equations $y = x^2 + ax + b$ and $y = x^2 + cx + d$, where a, b, c, and d are integers (not necessarily different), each chosen independently by rolling a fair six-sided die. What is the probability that the parabolas have at least one point in common?

(A) $\frac{1}{2}$ **(B)** $\frac{25}{36}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{31}{36}$ **(E)** 1

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14 Bernado and Silvia play the following game. An integer between 0 and 999, inclusive, is selected and given to Bernado. Whenever Bernado receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernado. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernado. What is the sum of the digits of N?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

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15 Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{\sqrt{10}}{10}$ **(D)** $\frac{\sqrt{5}}{6}$ **(E)** $\frac{\sqrt{10}}{5}$

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16 Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

(A) 108 **(B)** 132 **(C)** 671 **(D)** 846 **(E)** 1105

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17 Square $PQRS$ lies in the first quadrant. Points $(3,0)$, $(5,0)$, $(7,0)$, and $(13,0)$ lie on lines SP, RQ, PQ, and SR, respectively. What is the sum of the coordinates of the center of the square $PQRS$?

(A) 6 **(B)** 6.2 **(C)** 6.4 **(D)** 6.6 **(E)** 6.8

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18 Let $(a_1, a_2, \dots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

(A) 120 **(B)** 512 **(C)** 1024 **(D)** $181,440$ **(E)** $362,880$

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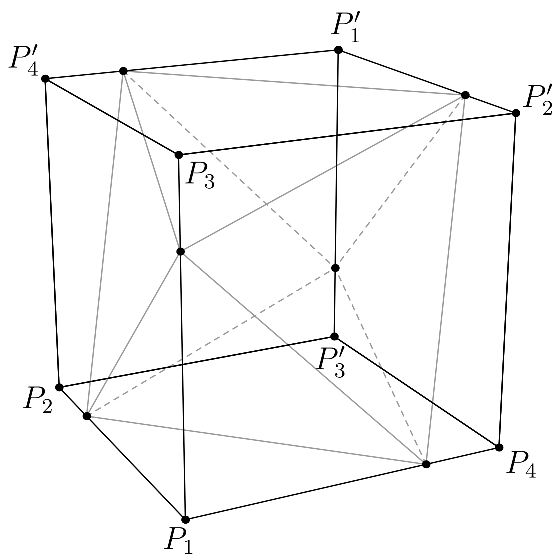
19 A unit cube has vertices $P_1, P_2, P_3, P_4, P_1', P_2', P_3'$, and P_4'. Vertices P_2, P_3, and P_4 are adjacent to P_1, and for $1 \le i \le 4$, vertices P_i and P_i' are opposite to each other. A regular octahedron has one vertex in each of the segments $P_1P_2, P_1P_3, P_1P_4, P_1'P_2', P_1'P_3'$, and $P_1'P_4'$. What is the octahedron's side length?



(A) $\frac{3\sqrt{2}}{4}$ **(B)** $\frac{7\sqrt{6}}{16}$ **(C)** $\frac{\sqrt{5}}{2}$ **(D)** $\frac{2\sqrt{3}}{3}$ **(E)** $\frac{\sqrt{6}}{2}$

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20 A trapezoid has side lengths $3, 5, 7$, and 11. The sum of all the possible areas of the trapezoid can be written in the form of $r_1\sqrt{n_1} + r_2\sqrt{n_2} + r_3$, where r_1, r_2, and r_3 are rational numbers and n_1 and n_2 are positive integers not divisible by the square of a prime. What is the greatest integer less than or equal to

$$r_1 + r_2 + r_3 + n_1 + n_2?$$

(A) 57 **(B)** 59 **(C)** 61 **(D)** 63 **(E)** 65

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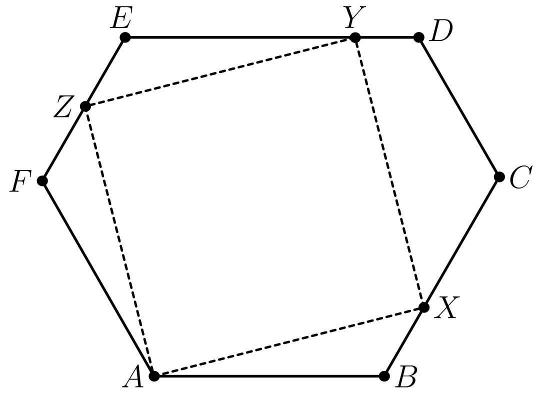
21 Square $AXYZ$ is inscribed in equiangular hexagon $ABCDEF$ with X on $\overline{BC}$, Y on $\overline{DE}$, and Z on $\overline{EF}$. Suppose that $AB = 40$, and $EF = 41(\sqrt{3} - 1)$. What is the side-length of the square?



(A) $29\sqrt{3}$ **(B)** $\frac{21}{2}\sqrt{2}+\frac{41}{2}\sqrt{3}$ **(C)** $20\sqrt{3}+16$ **(D)** $20\sqrt{2}+13\sqrt{3}$ **(E)** $21\sqrt{6}$

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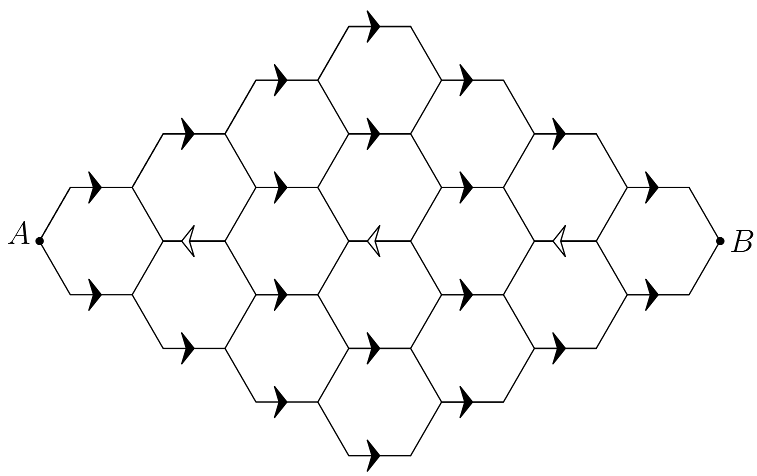
22 A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?



(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

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23 Consider all polynomials of a complex variable, $P(z)=4z^4+az^3+bz^2+cz+d$, where a,b,c and d are integers, $0\le d\le c\le b\le a\le 4$, and the polynomial has a zero z_0 with $|z_0|=1$. What is the sum of all values $P(1)$ over all the polynomials with these properties?

(A) 84 **(B)** 92 **(C)** 100 **(D)** 108 **(E)** 120

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24 Define the function f_1 on the positive integers by setting $f_1(1)=1$ and if $n=p_1^{e_1}p_2^{e_2}...p_k^{e_k}$ is the prime factorization of $n > 1$, then

$$f_1(n) = (p_1+1)^{e_1-1}(p_2+1)^{e_2-1}\cdots(p_k+1)^{e_k-1}.$$

For every $m \geq 2$, let $f_m(n) = f_1(f_{m-1}(n))$. For how many N in the range $1 \leq N \leq 400$ is the sequence $(f_1(N), f_2(N), f_3(N), ...)$ unbounded?

Note: a sequence of positive numbers is unbounded if for every integer B, there is a member of the sequence greater than B.

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

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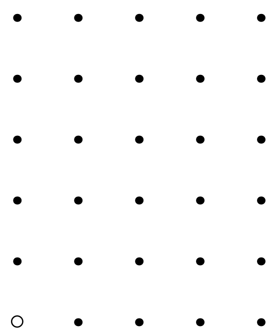
25 Let $S = \{(x, y) : x \in \{0, 1, 2, 3, 4\}, y \in \{0, 1, 2, 3, 4, 5\}$, and $(x, y) \neq (0, 0)\}$. Let T be the set of all right triangles whose vertices are in S. For every right triangle $t = \triangle ABC$ with vertices A, B, and C in counter-clockwise order and right angle at A, let $f(t) = \tan(\angle CBA)$. What is

$$\prod_{t \in T} f(t)?$$

S



(A) 1 **(B)** $\frac{625}{144}$ **(C)** $\frac{125}{24}$ **(D)** 6 **(E)** $\frac{625}{24}$
