**India National Olympiad 1987**

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**1** Given $m$ and $n$ as relatively prime positive integers greater than one, show that

$$\frac{\log_{10} m}{\log_{10} n}$$

is not a rational number.

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**2** Determine the largest number in the infinite sequence

$$1, \sqrt[2]{2}, \sqrt[3]{3}, \sqrt[4]{4}, \ldots, \sqrt[n]{n}, \ldots$$

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**3** Let $T$ be the set of all triplets $(a, b, c)$ of integers such that $1 \leq a < b < c \leq 6$ For each triplet $(a, b, c)$ in $T$, take number $a \cdot b \cdot c$. Add all these numbers corresponding to all the triplets in $T$. Prove that the answer is divisible by 7.

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**4** If $x$, $y$, $z$, and $n$ are natural numbers, and $n \geq z$ then prove that the relation $x^n + y^n = z^n$ does not hold.

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**5** Find a finite sequence of 16 numbers such that:
(a) it reads same from left to right as from right to left.
(b) the sum of any 7 consecutive terms is $-1$,
(c) the sum of any 11 consecutive terms is $+1$.

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**6** Prove that if coefficients of the quadratic equation $ax^2 + bx + c = 0$ are odd integers, then the roots of the equation cannot be rational numbers.

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**7** Construct the $\triangle ABC$, given $h_a$, $h_b$ (the altitudes from $A$ and $B$) and $m_a$, the median from the vertex $A$.

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**8** Three congruent circles have a common point $O$ and lie inside a given triangle. Each circle touches a pair of sides of the triangle. Prove that the incentre and the circumcentre of the triangle and the common point $O$ are collinear.

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**9** Prove that any triangle having two equal internal angle bisectors (each measured from a vertex to the opposite side) is isosceles.

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