

India National Olympiad 1994

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- 1 Let G be the centroid of the triangle ABC in which the angle at C is obtuse and AD and CF be the medians from A and C respectively onto the sides BC and AB . If the points B, D, G and F are concyclic, show that $\frac{AC}{BC} \geq \sqrt{2}$. If further P is a point on the line BG extended such that $AGCP$ is a parallelogram, show that triangle ABC and GAP are similar.

- 2 If $x^5 - x^3 + x = a$, prove that $x^6 \geq 2a - 1$.

- 3 In any set of 181 square integers, prove that one can always find a subset of 19 numbers, sum of whose elements is divisible by 19.

- 4 Find the number of nondegenerate triangles whose vertices lie in the set of points (s, t) in the plane such that $0 \leq s \leq 4, 0 \leq t \leq 4, s$ and t are integers.

- 5 A circle passes through the vertex of a rectangle $ABCD$ and touches its sides AB and AD at M and N respectively. If the distance from C to the line segment MN is equal to 5 units, find the area of rectangle $ABCD$.

- 6 Find all real-valued functions f on the reals such that $f(-x) = -f(x), f(x+1) = f(x) + 1$ for all x , and $f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2}$ for $x \neq 0$.