India National Olympiad 2000

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1 The incircle of ABC touches BC, CA, AB at K, L, M respectively. The line through A parallel to LK meets MK at P, and the line through A parallel to MK meets LK at Q. Show that the line PQ bisects AB and bisects AC.

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2 Solve for integers x, y, z:

$$\left\{ \begin{array}{rcl} x+y & = & 1-z \\ x^3+y^3 & = & 1-z^2. \end{array} \right.$$

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3 If a, b, c, x are real numbers such that $abc \neq 0$ and

$$\frac{xb+(1-x)c}{a} = \frac{xc+(1-x)a}{b} = \frac{xa+(1-x)b}{c},$$

then prove that $a = b = c$.

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4 In a convex quadrilateral $PQRS$, $PQ = RS$, $(\sqrt{3}+1)QR = SP$ and $\angle RSP - \angle SQP = 30^\circ$. Prove that $\angle PQR - \angle QRS = 90^\circ$.

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5 Let a, b, c be three real numbers such that $1 \geq a \geq b \geq c \geq 0$. prove that if λ is a root of the cubic equation $x^3 + ax^2 + bx + c = 0$ (real or complex), then $|\lambda| \leq 1.$

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6 For any natural numbers n, ($n \geq 3$), let $f(n)$ denote the number of congruent integer-sided triangles with perimeter n. Show that

(i) $f(1999) > f(1996)$;

(ii) $f(2000) = f(1997)$.

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