**India National Olympiad 2015**

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by TripteshBiswas

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| – | February 1st |
| **1** | Let $ABC$ be a right-angled triangle with $\angle B = 90^\circ$. Let $BD$ is the altitude from $B$ on $AC$. Let $P, Q$ and $I$be the incenters of triangles $ABD, CBD$ and $ABC$ respectively.Show that circum-center of triangle $PIQ$ lie on the hypotenuse $AC$. |
| **2** | For any natural number $n > 1$ write the finite decimal expansion of $\frac{1}{n}$ (for example we write $\frac{1}{2} = 0.4\overline{9}$ as its infinite decimal expansion not $0.5$). Determine the length of non-periodic part of the (infinite) decimal expansion of $\frac{1}{n}$. |
| **3** | Find all real functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(x^2 + yf(x)) = xf(x + y)$. |
| **4** | There are four basketball players $A, B, C, D$. Initially the ball is with $A$. The ball is always passed from one person to a different person.<br>In how many ways can the ball come back to $A$ after **seven** moves? (for example $A \to C \to B \to D \to A \to B \to C \to A$, or $A \to D \to A \to D \to C \to A \to B \to A$). |
| **5** | Let $ABCD$ be a convex quadrilateral.Let diagonals $AC$ and $BD$ intersect at $P$. Let $PE, PF, PG$ and $PH$ are altitudes from $P$ on the side $AB, BC, CD$ and $DA$ respectively. Show that $ABCD$ has a incircle if and only if $\frac{1}{PE} + \frac{1}{PG} = \frac{1}{PF} + \frac{1}{PH}$. |
| **6** | Show that from a set of $11$ square integers one can select six numbers $a^2, b^2, c^2, d^2, e^2, f^2$ such that $a^2 + b^2 + c^2 \equiv d^2 + e^2 + f^2 \pmod{12}$. |