Canada National Olympiad 1980

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by BigSams

1   If $a679b$ is the decimal expansion of a number in base 10, such that it is divisible by 72, determine a, b.

2   The numbers from 1 to 50 are printed on cards. The cards are shuffled and then laid out face up in 5 rows of 10 cards each. The cards in each row are rearranged to make them increase from left to right. The cards in each column are then rearranged to make them increase from top to bottom. In the final arrangement, do the cards in the rows still increase from left to right?

3   Among all triangles having (i) a fixed angle A and (ii) an inscribed circle of fixed radius r, determine which triangle has the least minimum perimeter.

4   A gambling student tosses a fair coin. She gains 1 point for each head that turns up, and gains 2 points for each tail that turns up. Prove that the probability of the student scoring *exactly* n points is $\frac{1}{3} \cdot \left(2 + \left(-\frac{1}{2}\right)^n\right)$.

5   A parallelepiped has the property that all cross sections, which are parallel to any fixed face F, have the same perimeter as F. Determine whether or not any other polyhedron has this property.

Typesetter's Note: I believe that proof of existence or non-existence suffices.