Canada National Olympiad 1983

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by BigSams

1 Find all positive integers w, x, y and z which satisfy $w! = x! + y! + z!$.

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2 For each $r \in \mathbb{R}$ let T_r be the transformation of the plane that takes the point (x, y) into the point $(2^r x; r2^r x + 2^r y)$. Let F be the family of all such transformations (i.e. $F = \{T_r : r \in \mathbb{R}\}$). Find all curves $y = f(x)$ whose graphs remain unchanged by every transformation in F.

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3 The area of a triangle is determined by the lengths of its sides. Is the volume of a tetrahedron determined by the areas of its faces?

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4 Prove that for every prime number p, there are infinitely many positive integers n such that p divides $2^n - n$.

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5 The geometric mean (G.M.) of k positive integers a_1, a_2, $\dots$, a_k is defined to be the (positive) k-th root of their product. For example, the G.M. of 3, 4, 18 is 6. Show that the G.M. of a set S of n positive numbers is equal to the G.M. of the G.M.'s of all non-empty subsets of S.
