Canada National Olympiad 1985

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1 The lengths of the sides of a triangle are 6, 8 and 10 units. Prove that there is exactly one straight line which simultaneously bisects the area and perimeter of the triangle.

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2 Prove or disprove that there exists an integer which is doubled when the initial digit is transferred to the end.

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3 Let P_1 and P_2 be regular polygons of 1985 sides and perimeters x and y respectively. Each side of P_1 is tangent to a given circle of circumference c and this circle passes through each vertex of P_2. Prove $x+y \geq 2c$. (You may assume that $\tan\theta \geq \theta$ for $0 \leq \theta < \frac{\pi}{2}$.)

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4 Prove that 2^{n-1} divides $n!$ if and only if $n = 2^{k-1}$ for some positive integer k.

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5 Let $1 < x_1 < 2$ and, for $n = 1$, 2, $\dots$, define $x_{n+1} = 1 + x_n - \frac{1}{2}x_n^2$. Prove that, for $n \geq 3$, $|x_n - \sqrt{2}| < 2^{-n}$.
