

Canada National Olympiad 1987

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- 1 Find all solutions of $a^2 + b^2 = n!$ for positive integers a, b, n with $a \leq b$ and $n < 14$.

- 2 The number 1987 can be written as a three digit number xyz in some base b . If $x + y + z = 1 + 9 + 8 + 7$, determine all possible values of x, y, z, b .

- 3 Suppose $ABCD$ is a parallelogram and E is a point between B and C on the line BC . If the triangles DEC, BED and BAD are isosceles what are the possible values for the angle DAB ?

- 4 On a large, flat field n people are positioned so that for each person the distances to all the other people are different. Each person holds a water pistol and at a given signal fires and hits the person who is closest. When n is odd show that there is at least one person left dry. Is this always true when n is even?

- 5 For every positive integer n show that

$$[\sqrt{4n+1}] = [\sqrt{4n+2}] = [\sqrt{4n+3}] = [\sqrt{n} + \sqrt{n+1}]$$

where $[x]$ is the greatest integer less than or equal to x (for example $[2.3] = 2, [\pi] = 3, [5] = 5$).