Canada National Olympiad 1988

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1 For what real values of k do $1988x^2 + kx + 8891$ and $8891x^2 + kx + 1988$ have a common zero?

2 A house is in the shape of a triangle, perimeter P metres and area A square metres. The garden consists of all the land within 5 metres of the house. How much land do the garden and house together occupy?

3 Suppose that S is a finite set of at least five points in the plane; some are coloured red, the others are coloured blue. No subset of three or more similarly coloured points is collinear. Show that there is a triangle
(i) whose vertices are all the same colour, and
(ii) at least one side of the triangle does not contain a point of the opposite colour.

4 Let $x_{n+1} = 4x_n - x_{n-1}$, $x_0 = 0$, $x_1 = 1$, and $y_{n+1} = 4y_n - y_{n-1}$, $y_0 = 1$, $y_1 = 2$. Show that for all $n \geq 0$ that $y_n^2 = 3x_n^2 + 1$.

5 If S is a sequence of positive integers let $p(S)$ be the product of the members of S. Let $m(S)$ be the arithmetic mean of $p(T)$ for all non-empty subsets T of S. Suppose that S' is formed from S by appending an additional positive integer. If $m(S) = 13$ and $m(S') = 49$, find S'.