Canada National Olympiad 1990

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1 A competition involving $n \geq 2$ players was held over k days. In each day, the players received scores of $1, 2, 3, \ldots, n$ points with no players receiving the same score. At the end of the k days, it was found that each player had exactly 26 points in total. Determine all pairs (n, k) for which this is possible.

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2 $\frac{n(n+1)}{2}$ distinct numbers are arranged at random into n rows. The first row has 1 number, the second has 2 numbers, the third has 3 numbers and so on. Find the probability that the largest number in each row is smaller than the largest number in each row with more numbers.

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3 The feet of the perpendiculars from the intersection point of the diagonals of a convex cyclic quadrilateral to the sides form a quadrilateral q. Show that the sum of the lengths of each pair of opposite sides of q is equal.

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4 A particle can travel at speeds up to $\frac{2m}{s}$ along the x-axis, and up to $\frac{1m}{s}$ elsewhere in the plane. Provide a labelled sketch of the region which can be reached within one second by the particle starting at the origin.

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5 The function $f : \mathbb{N} \to \mathbb{R}$ satisfies $f(1) = 1, f(2) = 2$ and

$$f(n+2) = f(n+2-f(n+1)) + f(n+1-f(n)).$$

Show that $0 \leq f(n+1) - f(n) \leq 1$. Find all n for which $f(n) = 1025$.
