Canada National Olympiad 1991

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by AwesomeToad, Amir Hossein

1 Show that the equation $x^2 + y^5 = z^3$ has infinitely many solutions in integers x, y, z for which $xyz \neq 0$.

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2 Let n be a fixed positive integer. Find the sum of all positive integers with the property that in base 2 each has exactly $2n$ digits, consisting of n 1's and n 0's. (The first digit cannot be 0.)

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3 Let C be a circle and P a given point in the plane. Each line through P which intersects C determines a chord of C. Show that the midpoints of these chords lie on a circle.

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4 Can ten distinct numbers $a_1, a_2, b_1, b_2, b_3, c_1, c_2, d_1, d_2, d_3$ be chosen from $\{0, 1, 2, \ldots, 14\}$, so that the 14 differences $|a_1 - b_1|$, $|a_1 - b_2|$, $|a_1 - b_3|$, $|a_2 - b_1|$, $|a_2 - b_2|$, $|a_2 - b_3|$, $|c_1 - d_1|$, $|c_1 - d_2|$, $|c_1 - d_3|$, $|c_2 - d_1|$, $|c_2 - d_2|$, $|c_2 - d_3|$, $|a_1 - c_1|$, and $|a_2 - c_2|$ are all distinct?

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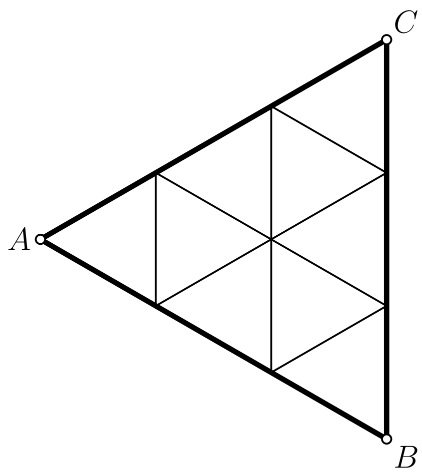
5 The sides of an equilateral triangle ABC are divided into n equal parts $(n \geq 2)$. For each point on a side, we draw the lines parallel to other sides of the triangle ABC, e.g. for $n = 3$ we have the following diagram:



For each $n \geq 2$, find the number of existing parallelograms.
