

Canada National Olympiad 1992

www.artofproblemsolving.com/community/c5037

by Silverfalcon

- 1 Prove that the product of the first n natural numbers is divisible by the sum of the first n natural numbers if and only if $n + 1$ is not an odd prime.
-

- 2 For $x, y, z \geq 0$, establish the inequality

$$x(x - z)^2 + y(y - z)^2 \geq (x - z)(y - z)(x + y - z)$$

and determine when equality holds.

- 3 In the diagram, $ABCD$ is a square, with U and V interior points of the sides AB and CD respectively. Determine all the possible ways of selecting U and V so as to maximize the area of the quadrilateral $PUQV$.

<http://i250.photobucket.com/albums/gg265/geometry101/CM01992Number3.jpg>

- 4 Solve the equation

$$x^2 + \frac{x^2}{(x + 1)^2} = 3$$

- 5 A deck of $2n + 1$ cards consists of a joker and, for each number between 1 and n inclusive, two cards marked with that number. The $2n + 1$ cards are placed in a row, with the joker in the middle. For each k with $1 \leq k \leq n$, the two cards numbered k have exactly $k - 1$ cards between them. Determine all the values of n not exceeding 10 for which this arrangement is possible. For which values of n is it impossible?
-