**Canada National Olympiad 1994**

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by BigSams

**1** Evaluate $\sum_{n=1}^{1994}\left((-1)^n\cdot\left(\frac{n^2+n+1}{n!}\right)\right)$.

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**2** Prove that $(\sqrt{2}-1)^n\ \forall n\in\mathbb{Z}^+$ can be represented as $\sqrt{m}-\sqrt{m-1}$ for some $m\in\mathbb{Z}^+$.

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**3** $25$ men sit around a circular table. Every hour there is a vote, and each must respond *yes* or *no*. Each man behaves as follows: on the $n^{\text{th}}$, vote if his response is the same as the response of at least one of the two people he sits between, then he will respond the same way on the $(n+1)^{\text{th}}$ vote as on the $n^{\text{th}}$ vote; but if his response is different from that of both his neighbours on the $n^{\text{th}}$ vote, then his response on the $(n+1)^{\text{th}}$ vote will be different from his response on the $n^{\text{th}}$ vote. Prove that, however everybody responded on the first vote, there will be a time after which nobody's response will ever change.

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**4** Let $AB$ be a diameter of a circle $\Omega$ and $P$ be any point not on the line through $AB$. Suppose that the line through $PA$ cuts $\Omega$ again at $U$, and the line through $PB$ cuts $\Omega$ at $V$. Note that in case of tangency, $U$ may coincide with $A$ or $V$ might coincide with $B$. Also, if $P$ is on $\Omega$ then $P=U=V$. Suppose that $|PU|=s|PA|$ and $|PV|=t|PB|$ for some $0\leq s,t\in\mathbb{R}$. Determine $\cos\angle APB$ in terms of $s,t$.

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**5** Let $ABC$ be an acute triangle. Let $AD$ be the altitude on $BC$, and let $H$ be any interior point on $AD$. Lines $BH,CH$, when extended, intersect $AC,AB$ at $E,F$ respectively. Prove that $\angle EDH=\angle FDH$.

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