Canada National Olympiad 1997

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by BigSams

1 Determine the number of pairs of positive integers x, y such that $x \le y$, $\gcd(x, y) = 5!$ and lcm$(x, y) = 50!$.

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2 The closed interval $A = [0, 50]$ is the union of a finite number of closed intervals, each of length 1. Prove that some of the intervals can be removed so that those remaining are mutually disjoint and have total length greater than 25.
Note: For reals $a \le b$, the closed interval $[a, b] := \{x \in \mathbb{R} : a \le x \le b\}$ has length $b - a$; disjoint intervals have *empty* intersection.

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3 Prove that $\frac{1}{1999} < \prod_{i=1}^{999} \frac{2i-1}{2i} < \frac{1}{44}$.

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4 The point O is situated inside the parallelogram $ABCD$ such that $\angle AOB + \angle COD = 180^\circ$. Prove that $\angle OBC = \angle ODC$.

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5 Write the sum $\sum_{i=0}^{n} \frac{(-1)^i \cdot \binom{n}{i}}{i^3+9i^2+26i+24}$ as the ratio of two explicitly defined polynomials with integer coefficients.
