Canada National Olympiad 2004

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1 Find all ordered triples (x,y,z) of real numbers which satisfy the following system of equations:

$$\begin{cases} xy &= z-x-y \\ xz &= y-x-z \\ yz &= x-y-z \end{cases}$$

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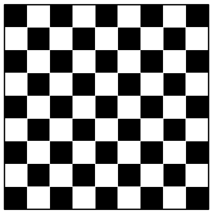
2 How many ways can 8 mutually non-attacking rooks be placed on the 9×9 chessboard (shown here) so that all 8 rooks are on squares of the same color?

(Two rooks are said to be attacking each other if they are placed in the same row or column of the board.)



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3 Let A,B,C,D be four points on a circle (occurring in clockwise order), with $AB<AD$ and $BC>CD$. The bisectors of angles BAD and BCD meet the circle at X and Y, respectively. Consider the hexagon formed by these six points on the circle. If four of the six sides of the hexagon have equal length, prove that BD must be a diameter of the circle.

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4 Let p be an odd prime. Prove that:

$$\sum_{k=1}^{p-1} k^{2p-1} \equiv \frac{p(p+1)}{2} \pmod{p^2}$$

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5 Let T be the set of all positive integer divisors of 2004^{100}. What is the largest possible number of elements of a subset S of T such that no element in S divides any other element in S?
