Germany Team Selection Test 1977

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by orl, Peter

1 We consider two sequences of real numbers $x_1 \geq x_2 \geq \ldots \geq x_n$ and $y_1 \geq y_2 \geq \ldots \geq y_n$. Let $z_1, z_2, \ldots, z_n$ be a permutation of the numbers $y_1, y_2, \ldots, y_n$. Prove that $\sum_{i=1}^{n}(x_i - y_i)^2 \leq \sum_{i=1}^{n}(x_i - z_i)^2$.

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2 Determine the polynomials P of two variables so that:

a.) for any real numbers t, x, y we have $P(tx, ty) = t^n P(x, y)$ where n is a positive integer, the same for all t, x, y;

b.) for any real numbers a, b, c we have $P(a+b, c) + P(b+c, a) + P(c+a, b) = 0$;

c.) $P(1, 0) = 1$.

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3 Let $a_1, \ldots, a_n$ be an infinite sequence of strictly positive integers, so that $a_k < a_{k+1}$ for any k. Prove that there exists an infinity of terms a_m, which can be written like $a_m = x \cdot a_p + y \cdot a_q$ with x, y strictly positive integers and $p \neq q$.

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4 When 4444^{4444} is written in decimal notation, the sum of its digits is A. Let B be the sum of the digits of A. Find the sum of the digits of B. (A and B are written in decimal notation.)
