Mathematical Olympiad Finals 1991

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1 Let $P,\ Q,\ R$ be the points such that $BP:PC=CQ:QA=AR:RB=t:1-t\ (0<t<1)$ for a triangle ABC.
Denote K by the area of the triangle with segments $AP,\ BQ,\ CR$ as side lengths and L by triangle ABC, find $\frac{K}{L}$ in terms of t.

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2 Let N be the set of the whole of positive integers. The mapping from N to N is defined as follows: $p(1)=2,\ p(2)=3,\ p(3)=4,\ p(4)=1,\quad p(n)=n\ (n\geq 5)$, $q(1)=3,\ q(2)=4,\ q(3)=2,\ q(4)=1,\quad p(n)=n\ (n\geq 5)$. Answer the following questions.

(1) If you make a mapping $f:N\rightarrow N$ sucessfully, we have f such that $f(f(n))=p(n)+2$. Give an example.

(2) Prove that it is impossible that $f(f(n))=q(n)+2$ holds in regardless of any definition for $f:N\rightarrow N$.

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3 Let A be a positive 16 digit integer.If you take out some consecutive digits integers among A, prove that we can make the product of the numbers be square number. For example if some digit of A is 4, you may take out only the digit.

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4 A rectangular of a $10*14$ is divided into small 140 unit squares and painted in red and white like chess board as below.
We put 0 or 1 in the square such that each row and column has an odd numbers of 1.
Prove that the number of 1 contained in red-painted square is even.

The pattern arranged by a red and a white square alternatively.

RWRWRWRW.....
WRWRWRWR.....
RWRWRWRW.....
WRWRWRWR.....

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5 Let A be a set of $n\geq 2$ points on a plane. Prove that there exists a circle which contains at least $\left[\frac{n}{3}\right]$ points of A among circles (involving perimeter) with some end points taken from A as the diameter, where $[x]$ is the greatest integer which is less than or equal to x.
