Mathematical Olympiad Finals 1992

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1 Let $x,\ y$ be relatively prime numbers such that $xy\neq 1$. For positive even integer n, prove that $x+y$ isn't a divisor of x^n+y^n.

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2 Suppose that $D,\ E$ are points on $AB,\ AC$ of $\triangle ABC$ with area 1 respectively and P is $BE\cap CD$. When $D,\ E$ move on $AB,\ AC$ with satisfying the condition $[BCED]=2\triangle PBC$, find the maximum area of $\triangle PDE$.

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3 Prove the inequality $\sum_{k=1}^{n-1}\frac{n}{n-k}\cdot\frac{1}{2^{k-1}}<4\ (n\geq 2)$.

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4 Let A be a $m\times n\ (m\neq n)$ matrix with the entries 0 and 1. Suppose that if f is injective such that $f\ :\ \{1,\ 2,\ \cdots,\ m\}\longrightarrow \{1,\ 2,\ \cdots,\ n\}$, then there exists $1\leq i\leq m$ such that $(i,\ f(i))$ element is 0.

Prove that there exist $S\subseteq \{1,\ 2,\ \cdots,\ m\}$ and $T\subseteq \{1,\ 2,\ \cdots,\ n\}$ satisfying the condition:
$i)$ if $i\in S,\ j\in T$, then $(i,\ j)$ entry is 0.
$ii)\ card\ S+card\ T>n.$

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5 Suppose that $n\geq 2$ be integer and $a_1,\ a_2,\ a_3,\ a_4$ satisfy the following condition:

i) n and $a_i\ (i=1,\ 2,\ 3,\ 4)$ are relatively prime.

ii) $(ka_1)_n+(ka_2)_n+(ka_3)_n+(ka_4)_n=2n$ for $k=1,\ 2,\ \cdots,\ n-1.$

Note that $(a)_n$ expresses the divisor when a is divided by n.

Prove that $(a_1)_n,\ (a_2)_n,\ (a_3)_n,\ (a_4)_n$ can be divided into two pair with sum n.
