Mathematical Olympiad Finals 1993

www.artofproblemsolving.com/community/c5078

by Kunihiko_Chikaya

1 Call a **word** forming by alphabetical small letters $a, b, c, \cdots x, y, z$ and **a periodic word** arranging by a certain word more than two times repeatedly.For example kyonkyon is eight-letter periodic word. Given two words $W_1,\ W_2$ which have the same number of letters and have a different first letter,
if you will remove the letter, W_1 and W_2 will be same word.Prove that either W_1 or W_2 is not periodic.

---

2 Denote by $d(n)$ the largest odd divisor of positive integers n and define $D(n),\ T(n)$ as follows.

$$D(n)=d(1)+d(2)+\cdots+d(n),\ \ T(n)=1+2+\cdots+n.$$

Prove that there existed infinitely positive integers n such that $3D(n)=2T(n)$.

---

3 x students took an exam with y problems. Each student solved a half of the problem and the number of person those who solved the problem is equal for each problem.For any two students exactly three problems could be solved by the two persons. Find all pairs of $(x,\ y)$ satisfying this condition, then show an example to prove that it is possiible the case in terms of $a,\ b$ as follows:
Note that every student solve (a) or don't solve (b).

---

4 Given five radii $l_1,\ \cdots l_5$ of a sphere S, no three of these radii are on a same plane.
Choose a pair to endpoint from each radius $l_1,\ \cdots l_5$. Find the number of choices such that five points are in a hemisphere among 32 choices of an endpoint.

---

5 Prove that there existed a positive number C, irrelevant to n and $a_1,\ a_2,\ \cdots a_n$, satisfying the following condition.

Condition: For arbiterary positive numbers n and arbiterary real numbers $a_1,\ \cdots a_n$, the following inequality holds.

$$\max_{0\leq x\leq 2}\prod_{j=1}^n |x-a_j|\leq C^n\max_{0\leq x\leq 1}\prod_{j=1}^n |x-a_j|.$$
