Mathematical Olympiad Finals 1997

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1 Take 10 points inside the circle with diameter 5. Prove that for any these 10 points there exist two points whose distance is less than 2.

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2 Prove that

$$\frac{(b+c-a)^2}{(b+c)^2+a^2}+\frac{(c+a-b)^2}{(c+a)^2+b^2}+\frac{(a+b-c)^2}{(a+b)^2+c^2}\geq\frac{3}{5}$$

for any positive real numbers a, b, c.

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3 Call the Graph the set which composed of several vertices $P_1,\cdots P_2$ and several edges $($segments$)$ connecting two points among these vertices. Now let G be a graph with 9 vertices and satisfies the following condition.

Condition: Even if we select any five points from the vertices in G, there exist at least two edges whose endpoints are included in the set of 5 points.

What is the minimum possible numbers of edges satisfying the condition?

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4 Let A,B,C,D be points in space which are not on a same plane and no any 3 points are not colinear.
Suppose that the sum of the segments $AX+BX+CX+DX$ is minimized at $X=X_0$ which is different from A,B,C,D. Prove that $\angle AX_0B=\angle CX_0D$.

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5 The letters A or B are assigned on the points divided equally into 2^n $(n=1,\ 2,\cdots)$ parts of a circumference.If you choose n letters from any succesively arranging points directed clockwise, prove that there exists the way of assignning for which the line of letters are mutually distinct.
