Mathematical Olympiad Finals 1999

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1 One can place a stone on each of the squares of a 1999×1999 board. Find the minimum number of stones that must be placed so that, for any blank square on the board, the total number of stones placed in the corresponding row and column is at least 1999.

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2 Let $f(x) = x^3 + 17$. Prove that for every integer $n \geq 2$ there exists a natural number x for which $f(x)$ is divisible by 3^n but not by 3^{n+1}.

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3 From a group of $2n+1$ weights, if we remove any weight, the remaining $2n$, can be divided in two groups of n elements, such that they have the same total weight. Prove all weights are equal.

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4 Prove that the polynomial $f(x) = (x^2+1)(x^2+2^2)\cdots(x^2+n^2)+1$ cannot be expressed as a product of two polynomials with integer coefficients with degree greater than 1.

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5 All sides of a convex hexagon $ABCDEF$ are 1. Let M, m be the maximum and minimum possible values of three diagonals AD, BE, CF. Find the range of M, m.
