Mathematical Olympiad Finals 2003

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by dragonfire

1 A point P lies in $\triangle ABC$. The lines BP, CP meet AC, AB at Q, R respectively. Given that $AR = RB = CP, CQ = PQ$, find $\angle BRC$.

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2 We have two distinct positive integers a, b with $a|b$. Each of a, b consists of $2n$ decimal digits. The first n digits of a are identical to the last n digits of b, and vice versa. Determine a, b.

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3 Find the greatest real number k such that, for any positive a, b, c with $a^2 > bc$, $(a^2 - bc)^2 > k(b^2 - ca)(c^2 - ab)$.

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4 Let $p, q \geq 2$ be coprime integers. A list of integers $(r, a_1, a_2, ..., a_n)$ with $|a_i| \geq 2$ for all i is said to be an expansion of p/q if $\frac{p}{q} = r + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{... + \frac{1}{a_n}}}}$.

Now define the weight of an expansion $(r, a_1, a_2, ..., a_n)$ to be the product $(|a_1| - 1)(|a_2| - 1)...(|a_n| - 1)$.

Show that the sum of the weights of all expansions of p/q is q.

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5 Find the greatest possible integer n such that one can place n points in a plane with no three on a line, and color each of them either red, green, or yellow so that:

(i) inside each triangle with all vertices red there is a green point.

(ii) inside each triangle with all vertices green there is a yellow point.

(iii) inside each triangle with all vertices yellow there is a red point.
