Mathematical Olympiad Finals 2005

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1 Double-faced coins are arranged with all the heads facing upward in the shape of 17×17.At one operation, you turn over 5 consecutive coins in longitudinal or 5 ones in transversal or 5 ones in oblique at the same time.Now can you make all those reverses face upward when you repeat this operation?

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2 Let $P(x,y), Q(x,y)$ be two-variable polynomials with the coefficients of integer.Supposed that when a_n, b_n are determined for certain integers $a_0,\ b_0$ by $a_{n+1} = P(a_n, b_n),\ b_{n+1} = Q(a_n, b_n)\ (n = 0, 1, 2, \cdots)$ there existed positive integer k such that $(a_1, b_1) \neq (a_0, b_0)$ and $(a_k, b_k) = (a_0, b_0)$.Prove that the number of the lattice points on the segment with end points of (a_n, b_n) and (a_{n+1}, b_{n+1}) is indepedent of n.

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3 Let a, b, c be positive real numbers such that $a + b + c = 1$.Prove the following inequality.

$$a\sqrt[3]{1+b-c} + b\sqrt[3]{1+c-a} + c\sqrt[3]{1+a-b} \leqq 1$$

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4 Given two points A and B on a circle Γ. Let the tangents to this circle Γ at the points A and B meet at a point X. Let C and D be two points on the circle Γ such that the points C, D, X are collinear in this order and such that the lines CA and BD are perpendicular.

Let the line CA intersect the line BD at a point F.
Let the line CD intersect the line AB at a point G.
Let H be the point of intersection of the segment BD and the perpendicular bisector of the segment GX.

Prove that the four points X, F, G, H lie on one circle.

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5 You are the boss. You have ten men and there are ten tasks. Your men have two parameters to each task, one is **enthusiasm**, the other is **ability**.Now you are to assign the tasks to your men one by one.When man A has more enthusiasm about task v than about task u, and man A has better ability in task v than man B does, though if you assign task u to man A and task v to man B, man A feel unsatisfied.Or, if there is a more efficient way than yours that you can assign each task to men with better ability, you will be brought to account by your employer.Prove that there is a way to assign tasks without any dissatisfaction or disadvantage.
