Mathematical Olympiad Finals 2009

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1 Find all positive integers n such that $8^n + n$ is divisible by $2^n + n$.

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2 Let N be postive integer. Some integers are written in a black board and those satisfy the following conditions.

1. Any numbers written are integers which are from 1 to N.

2. More than one integer which is from 1 to N is written.

3. The sum of numbers written is even.

If we mark X to some numbers written and mark Y to all remaining numbers, then prove that we can set the sum of numbers marked X are equal to that of numbers marked Y.

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3 Let $k \geq 2$ be integer, $n_1,\ n_2,\ n_3$ be positive integers and $a_1,\ a_2,\ a_3$ be integers from $1,\ 2,\ \cdots,\ k-1$.

Let $b_i = a_i \sum_{j=0}^{n_i} k^j\ (i = 1,\ 2,\ 3)$. Find all possible pairs of integers $(n_1,\ n_2,\ n_3)$ such that $b_1 b_2 = b_3$.

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4 Let Γ be a circumcircle. A circle with center O touches to line segment BC at P and touches the arc BC of Γ which doesn't have A at Q. If $\angle BAO = \angle CAO$, then prove that $\angle PAO = \angle QAO$.

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5 Find all functions f, defined on the non negative real numbers and taking non negative real numbers such that $f(x^2) + f(y) = f(x^2 + y + xf(4y))$ for any non negative real numbers $x,\ y$.
