Mathematical Olympiad Finals 2011

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1	Given an acute triangle ABC with the midpoint M of BC. Draw the perpendicular HP from the orthocenter H of ABC to AM. Show that $AM \cdot PM = BM^2$.
2	Find all of quintuple of positive integers (a, n, p, q, r) such that $a^n - 1 = (a^p - 1)(a^q - 1)(a^r - 1)$.
3	Person A writes down non negative integers in each N grid running in a line horizontally. When A says one non negative integer, Person B replaces some number in N grid by the number that A said. Repeat this procedure, when these numbers are arranged in the order of monotone increasing in the wider sense, the procedure is over. Is it possible that B can finish in regard less of A?
4	Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(f(x) - f(y)) = f(f(x)) - 2x^2 f(y) + f(y^2)$ for all $x, y \in \mathbb{R}$.
5	Given 4 points on a plane. Suppose radii of 4 incircles of the triangles, which can be formed by any 3 points taken from the 4 points, are equal. Prove that all of the triangles are congruent.