Mathematical Olympiad Finals 2014

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1	Let O be the circumcenter of triangle ABC, and let l be the line passing through the midpoint of segment BC which is also perpendicular to the bisector of angle $\angle BAC$. Suppose that the midpoint of segment AO lies on l. Find $\angle BAC$.
2	Find all ordered triplets of positive integers $(a,\ b,\ c)$ such that $2^a+3^b+1=6^c$.
3	In a school, there are n students and some of them are friends each other. (Friendship is mutual.) Define a, b the minimum value which satisfies the following conditions: (1) We can divide students into a teams such that two students in the same team are always friends. (2) We can divide students into b teams such that two students in the same team are never friends. Find the maximum value of $N=a+b$ in terms of n.
4	Let Γ be the circumcircle of triangle ABC, and let l be the tangent line of Γ passing A. Let D, E be the points each on side AB, AC such that $BD : DA = AE : EC$. Line DE meets Γ at points F, G. The line parallel to AC passing D meets l at H, the line parallel to AB passing E meets l at I. Prove that there exists a circle passing four points F, G, H, I and tangent to line BC.
5	Find the maximum value of real number k such that $$\frac{a}{1+9bc+k(b-c)^2}+\frac{b}{1+9ca+k(c-a)^2}+\frac{c}{1+9ab+k(a-b)^2}\geq \frac{1}{2}$$ holds for all non-negative real numbers $a,\ b,\ c$ satisfying $a+b+c=1$.

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