Dutch Mathematical Olympiad 1998

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1 Consider any permutation σ of $\{0,1,2,\dots,9\}$ and for each of the 8 triples of consecutive numbers in this permutation, consider the sum of these three numbers. Let $M(\sigma)$ be the largest of these 8 sums. (For example, for the permutation $\sigma=(4,6,2,9,0,1,8,5,7,3)$ we get the 8 sums 12, 17, 11, 10, 9, 14, 20, 15, and $M(\sigma)=20$.)

(a) Find a permutation σ_1 such that $M(\sigma_1)=13$.

(b) Does there exist a permutation σ_2 such that $M(\sigma_2)=12$?

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2 Let $TABCD$ be a pyramid with top vertex T, such that its base $ABCD$ is a square of side length 4. It is given that, among the triangles TAB, TBC, TCD and TDA, one can find an isosceles triangle and a right-angled triangle. Find all possible values for the volume of the pyramid.

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3 Let m and n be positive integers such that $m-n=189$ and such that the least common multiple of m and n is equal to 133866. Find m and n.

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4 Let $ABCD$ be a convex quadrilateral such that $AC\perp BD$.

(a) Prove that $AB^2+CD^2=BC^2+DA^2$.

(b) Let $PQRS$ be a convex quadrilateral such that $PQ=AB$, $QR=BC$, $RS=CD$ and $SP=DA$. Prove that $PR\perp QS$.

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5 Find all real solutions of the following equation:

$$(x+1995)(x+1997)(x+1999)(x+2001)+16=0.$$
