Brazil National Olympiad 1990

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1 Show that a convex polyhedron with an odd number of faces has at least one face with an even number of edges.

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2 There exists infinitely many positive integers such that $a^3 + 1990b^3 = c^4$.

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3 Each face of a tetrahedron is a triangle with sides a, b,c and the tetrahedon has circumradius 1. Find $a^2 + b^2 + c^2$.

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4 $ABCD$ is a quadrilateral, E, F, G, H are midpoints of AB, BC, CD, DA.
Find the point P such that $area(PHAE) = area(PEBF) = area(PFCG) = area(PGDH)$.

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5 Let $f(x) = \frac{ax+b}{cx+d}$ $F_n(x) = f(f(f...f(x)...))$ (with n $f's$)

Suppose that $f(0) \neq 0$, $f(f(0)) \neq 0$, and for some n we have $F_n(0) = 0$,
show that $F_n(x) = x$ (for any valid x).
