Brazil National Olympiad 1992

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Day 1

1 The equation $x^3 + px + q = 0$ has three distinct real roots. Show that $p < 0$

2 Show that there is a positive integer n such that the first 1992 digits of n^{1992} are 1.

3 Given positive real numbers $x_1, x_2, \dots, x_n$ find the polygon $A_0A_1\dots A_n$ with $A_iA_{i+1} = x_{i+1}$ and which has greatest area.

4 Let ABC be a triangle. Find the point D on its side AC and the point E on its side AB such that the area of triangle ADE equals to the area of the quadrilateral $DEBC$, and the segment DE has minimum possible length.

Day 2

5 Let $d(n) = \sum_{0<d|n} 1$. Show that, for any natural $n > 1$,

$$\sum_{2\leq i\leq n} \frac{1}{i} \leq \sum \frac{d(i)}{n} \leq \sum_{1\leq i\leq n} \frac{1}{i}$$

6 Given a set of n elements, find the largest number of subsets such that no subset is contained in any other

7 Find all 4-tuples (a, b, c, n) of naturals such that
$n^a + n^b = n^c$

8 In a chess tournament each player plays every other player once. A player gets 1 point for a win, 0.5 point for a draw and 0 for a loss. Both men and women played in the tournament and each player scored the same total of points against women as against men. Show that the total number of players must be a square.