**Brazil National Olympiad 1993**

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by Johann Peter Dirichlet

**1** The sequence $(a_n)_{n\in\mathbb{N}}$ is defined by $a_1 = 8, a_2 = 18, a_{n+2} = a_{n+1}a_n$. Find all terms which are perfect squares.

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**2** A real number with absolute value less than $1$ is written in each cell of an $n\times n$ array, so that the sum of the numbers in each $2\times 2$ square is zero. Show that for odd $n$ the sum of all the numbers is less than $n$.

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**3** Given a circle and its center $O$, a point $A$ inside the circle and a distance $h$, construct a triangle $BAC$ with $\angle BAC = 90^\circ$, $B$ and $C$ on the circle and the altitude from $A$ length $h$.

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**4** $ABCD$ is a convex quadrilateral with

$$\angle BAC = 30^\circ$$

$$\angle CAD = 20^\circ$$

$$\angle ABD = 50^\circ$$

$$\angle DBC = 30^\circ$$

If the diagonals intersect at $P$, show that $PC = PD$.

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**5** Find at least one function $f:\mathbb{R}\to\mathbb{R}$ such that $f(0) = 0$ and $f(2x+1) = 3f(x)+5$ for any real $x$.

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