Brazil National Olympiad 1999

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Day 1

1 Let $ABCDE$ be a regular pentagon. The star $ACEBD$ has area 1. AC and BE meet at P, while BD and CE meet at Q. Find the area of $APQD$.

2 Show that, if $\sqrt{2}$ is written in decimal notation, there is at least one nonzero digit at the interval of 1,000,000-th and 3,000,000-th digits.

3 How many coins can be placed on a 10×10 board (each at the center of its square, at most one per square) so that no four coins form a rectangle with sides parallel to the sides of the board?

Day 2

4 On planet Zork there are some cities. For every city there is a city at the diametrically opposite point. Certain roads join the cities on Zork. If there is a road between cities P and Q, then there is also a road between the cities P' and Q' diametrically opposite to P and Q. In plus, the roads do not cross each other and for any two cities P and Q it is possible to travel from P to Q.

The prices of Kriptonita in Urghs (the planetary currency) in two towns connected by a road differ by at most 100. Prove that there exist two diametrically opposite cities in which the prices of Kriptonita differ by at most 100 Urghs.

5 There are n football teams in *Tumbolia*. A championship is to be organised in which each team plays against every other team exactly once. Ever match takes place on a sunday and each team plays at most one match each sunday. Find the least possible positive integer m_n for which it is possible to set up a championship lasting m_n sundays.

6 Given any triangle ABC, show how to construct A' on the side AB, B' on the side BC and C' on the side CA, such that ABC and $A'B'C'$ are similar (with $\angle A = \angle A', \angle B = \angle B', \angle C = \angle C'$) and $A'B'C'$ has the least possible area.