Brazil National Olympiad 2002

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Day 1

1 Show that there is a set of 2002 distinct positive integers such that the sum of one or more elements of the set is never a square, cube, or higher power.

2 $ABCD$ is a cyclic quadrilateral and M a point on the side CD such that ADM and $ABCM$ have the same area and the same perimeter. Show that two sides of $ABCD$ have the same length.

3 The squares of an $m \times n$ board are labeled from 1 to mn so that the squares labeled i and $i+1$ always have a side in common. Show that for some k the squares k and $k+3$ have a side in common.

Day 2

4 For any non-empty subset A of $\{1, 2, \ldots, n\}$ define $f(A)$ as the largest element of A minus the smallest element of A. Find $\sum f(A)$ where the sum is taken over all non-empty subsets of $\{1, 2, \ldots, n\}$.

5 A finite collection of squares has total area 4. Show that they can be arranged to cover a square of side 1.

6 Show that we cannot form more than 4096 binary sequences of length 24 so that any two differ in at least 8 positions.